1.24 Polynomial Graphs

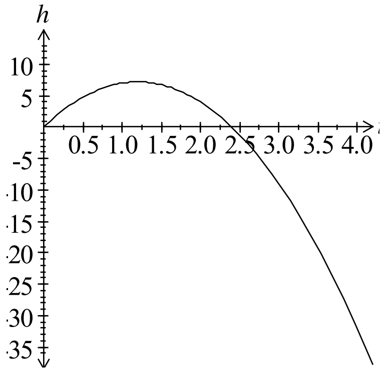
- Some pupils may think that all graphs that come from algebra are straight lines and that you can't get a curve from an algebra equation. It may be an eye-opener to see that maths can produce graphs that look like "real-life" graphs with all their bumps and turns. This makes for a good opportunity to discuss the principles of mathematical modelling.

1.24.1 Mathematical Modelling. Projectiles.

The equation $h = 12t - 5t^2$ models the motion of a stone thrown off the top of a cliff (ignoring air resistance, size and shape of stone, curvature of the earth, Einstein's theory of relativity, etc.).
h is the height of the stone in metres above the top of the cliff at time t seconds after letting go of it.
Draw a graph of h against t for the first 4 seconds (do a table of numbers first).

1. When does the stone reach the maximum height?
2. What is the maximum height?
3. When is the stone level with the cliff top?
4. The bottom of the cliff drop is 26 m. When does the ball land?
5. When is it going fastest?

Use 1 mm × 1 mm squared graph paper



Answers:

1. *After 1.2 s*
2. *7.2 m*
3. *After 2.4 s*
4. *After 3.78 s (2 dp)*
5. *As it's just about to hit the bottom (gradient has largest value – although negative); the vertical speed on impact is about 26 m/s.*

1.24.2 Pupils could investigate how the gradients of curved graphs vary as x varies.
Could try to draw the gradient functions on a sheet of different functions $f(x)$ (see sheet).

This could lead to differential calculus (see several sheets that do this with $y = x^2$ and $y = x^3$).

1.24.3 **NEED** a computer graph-plotting program.
Investigate the shape of the graphs of these functions with different numbers instead of the letters a and b.
Try a positive/negative and b positive/negative.
Sketch the results in a table (see right) for each function.
Could split up the class so that different pupils/groups cover different functions.

$y = ax + b$

$y = ax^2 + b$

$y = a\sqrt{x} + b$

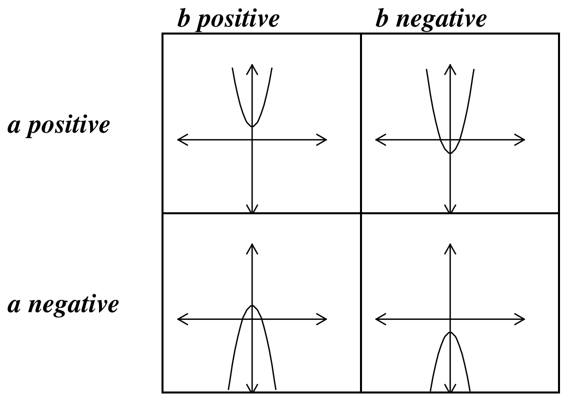
$y = ax^3 + b$

$y = \dfrac{a}{x+b}$

$y = x^a$, $a > 3$

$y = a^x$

Answers:
e.g., for $y = ax^2 + b$,

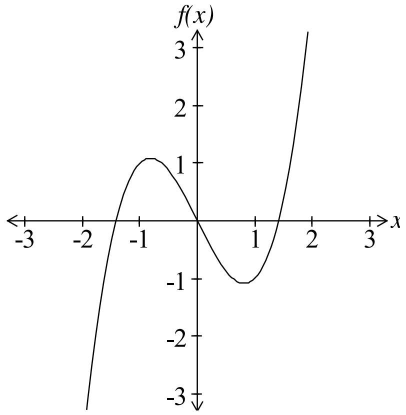


1.24.4 People graphs. Everyone stand up.
I want you to be $y = 0$ (arms out horizontally). Now be $y = x$, $y = -x$ (aeroplanes!), $y = x^2$, $y = -x^2$, $y = x^3$, etc.

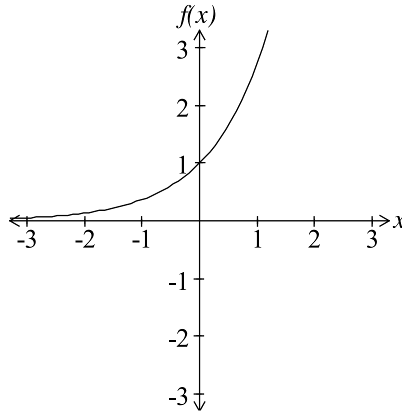
An interesting way of reviewing knowledge of the shapes of graphs!

To do $y = \sin x$ requires team-work!

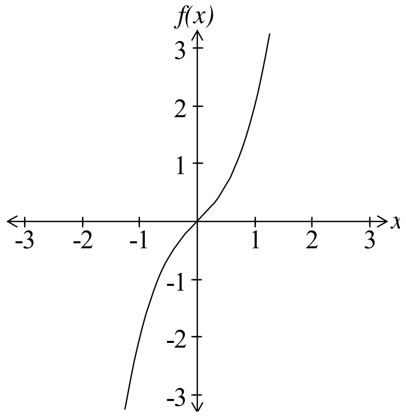
$f(x) = x^3 - 2x$



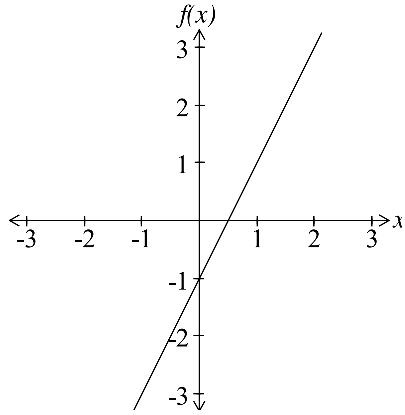
$f(x) = e^x$



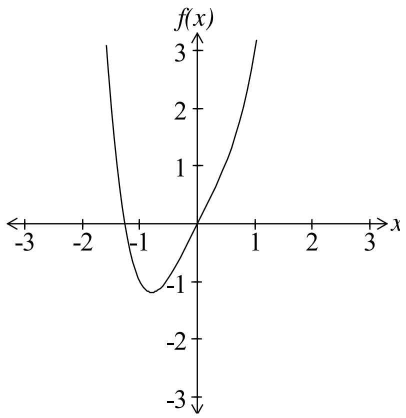
$f(x) = x^3 + x$



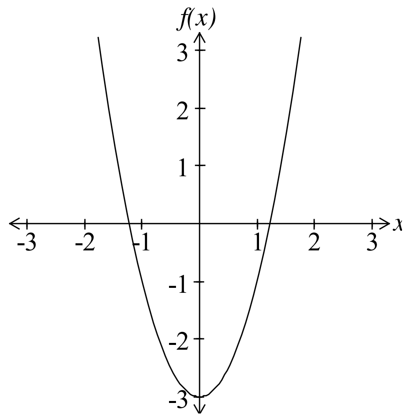
$f(x) = 2x - 1$



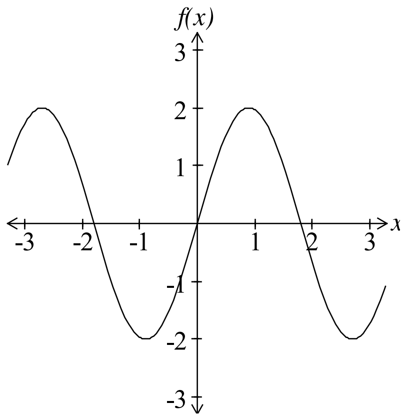
$f(x) = x^4 + 2x$



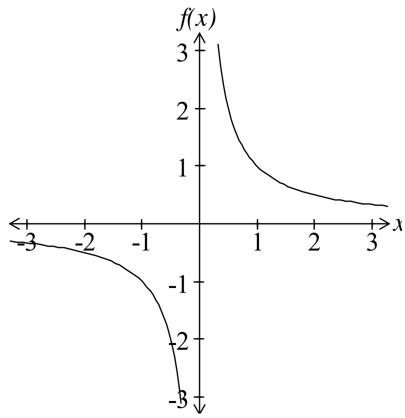
$f(x) = 2x^2 - 3$



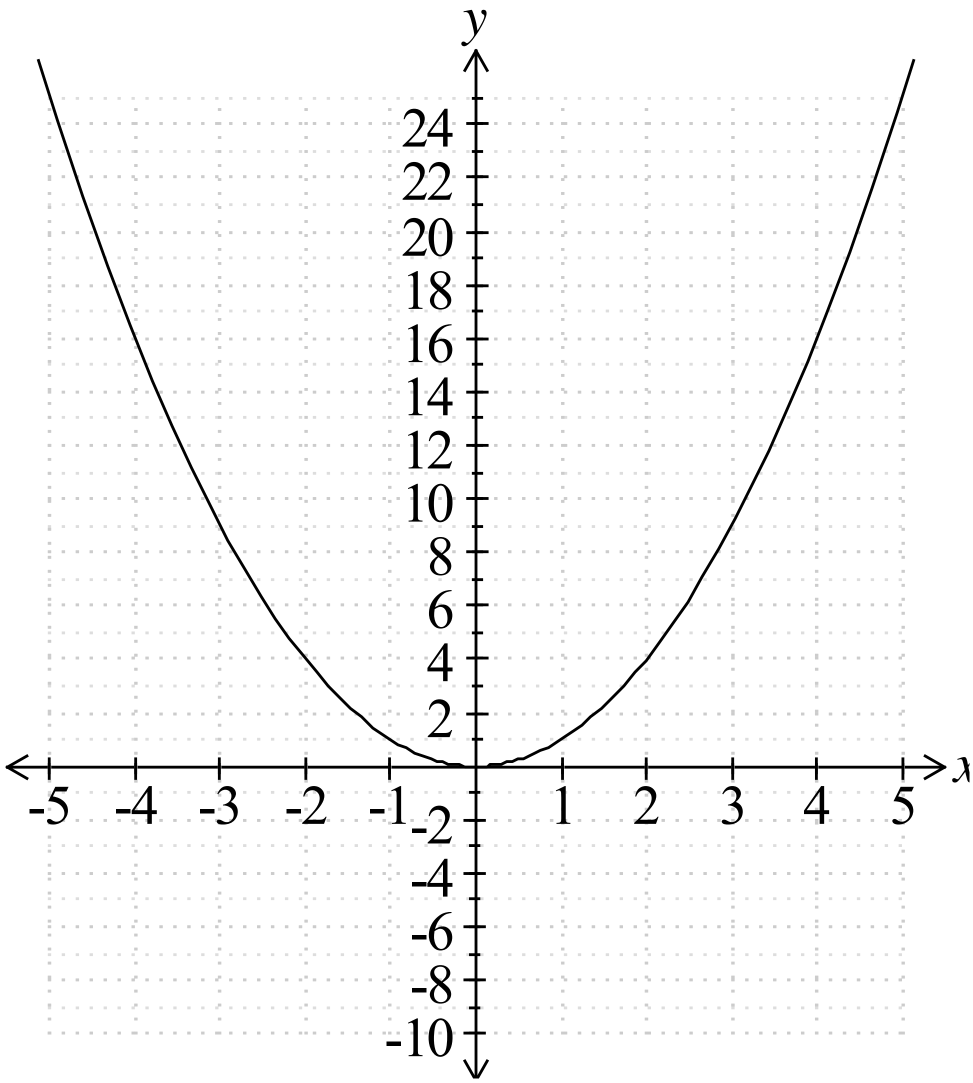
$f(x) = 2\sin 100x$



$f(x) = \dfrac{1}{x}$



Finding the gradient function for $y = x^2$



Remember that the gradient of a straight line between two points A (x_A, y_A) and B (x_B, y_B) is $\frac{\Delta y}{\Delta x} = \frac{y_B - y_A}{x_B - x_A}$.

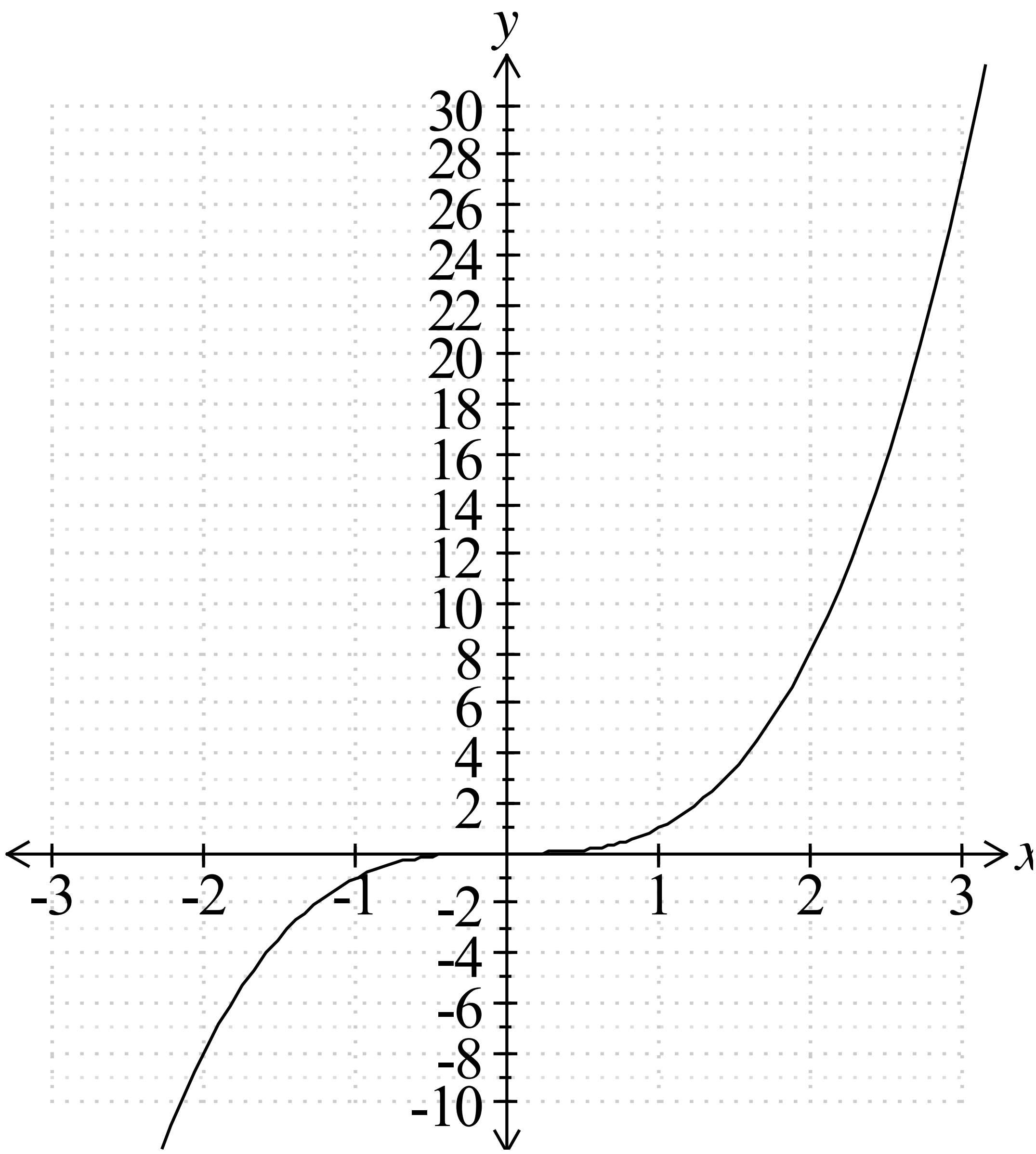
Fill in the table on the next page.
Then plot the *gradient function* on the graph above.
What is the equation of the gradient function graph?

Can you explain your results?

Finding the gradient function for $y = x^2$

point	nearby point	gradient	best gradient
(−5, 25)	(−4.9, 24.01)	−9.9	−10
	(−4.99,)		
	(−4.999,)		
(−4, 16)	(−3.9,)		
	(−3.99,)		
	(,)		
(−3,)	(,)		
	(,)		
	(,)		
(−2,)	(,)		
	(,)		
	(,)		
(−1,)	(,)		
	(,)		
	(,)		
(0, 0)	(0.1, 0.01)		
	(0.01,)		
	(,)		
(1,)	(1.1,)		
	(1.01,)		
	(,)		
(2,)	(2.1,)		
	(,)		
	(,)		
(3,)	(,)		
	(,)		
	(,)		
(4,)	(,)		
	(,)		
	(,)		
(5,)	(,)		
	(,)		
	(,)		

Finding the gradient function for $y = x^3$



Fill in the table on the next page.
Then plot the *gradient function* on the graph above.
What is the equation of the gradient function graph?

Can you explain your results?

Finding the gradient function for $y = x^3$

point	nearby point	gradient	best gradient
(−3, −27)	(−2.9, −24.39)	26.11	27
	(−2.99,)		
	(−2.999,)		
(−2,)	(−1.9,)		
	(,)		
	(,)		
(−1,)	(,)		
	(,)		
	(,)		
(0,)	(0.1,)		
	(,)		
	(,)		
(1,)	(,)		
	(,)		
	(,)		
(2,)	(,)		
	(,)		
	(,)		
(3,)	(,)		
	(,)		
	(,)		