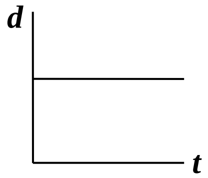
1.25 Real-Life Graphs

- A very valuable topic for understanding what a graph is/does/means. Possible contexts include distance-time (stones, cars, trains, etc.) and volume-time (water in pipes, etc.).
- It's worth trying to distinguish between *dependent* and *independent* variables. Which kind one is depends on the context. Sometimes it's an arbitrary choice. It's really a cause-and-effect relationship – the independent variable "causes" a change in the dependent one. If I go down the motorway at a constant 70 mph, how far I get (distance is the dependent variable) depends on time (the independent variable), but if I have to travel a fixed distance (say to London) then how long it takes me (time is the *dependent* variable this time) depends on my average speed (the independent variable).

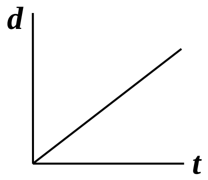
1.25.1 You could start with distance-time graphs.
"This is me in my car. What's happening?"

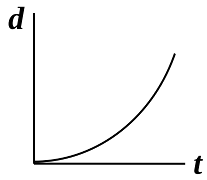


Answer:
"Run out of petrol", etc.

Expect "going along a flat road" and similar mistakes. Key questions are "how far have I got at this point?"; "how far have I got 1 minute later?". Put some numbers on the axes.

What's the difference between these two?

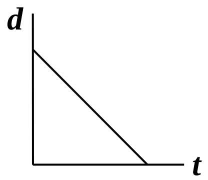




Steady speed versus accelerating.

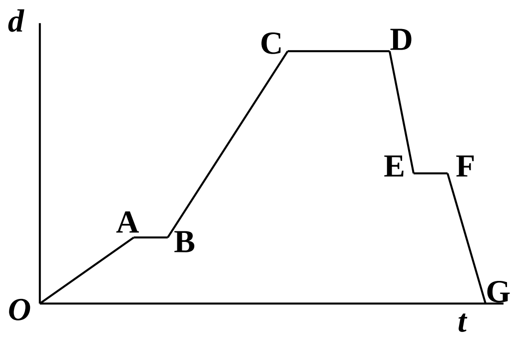
What would decelerating look like?
Answer: the curve would be reflected in the straight line; still going through the origin and distance increasing, but the slope getting shallower and shallower.

What's going on here?



Steady speed in the opposite direction.
Not necessarily downhill!

Draw on the board a record of a journey in my car (e.g., below). Pupils invent a story to fit it.
They can put letters in brackets as they write to indicate where I am along the graph at that point.



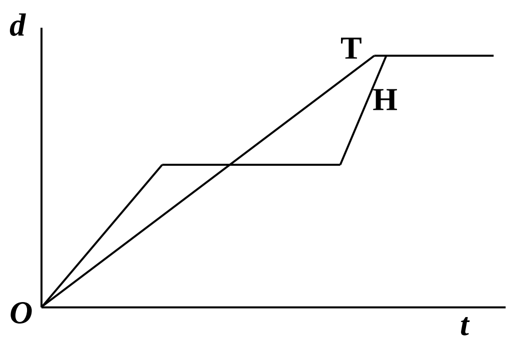
Stories often involve getting stopped by the police for speeding, so indicate in advance if you don't want this kind of humour!

Questions to sharpen up stories:
"When was I going fastest?" "Does that fit your story?" "Did I stop at the same place or a different place on the way back?" "If A to B was a traffic lights stop could I really have done my weekly shopping between C and D?!"
(You could allow pupils to modify their graph rather than re-write their whole story!)

Extension: Add scales to your drawing and work out the speeds at the different points.
Draw a speed-time or a velocity-time graph for the journey.

1.25.2 Draw a distance-time graph for the story of the hare and the tortoise. (You could ask one pupil to remind the rest of the story.)

(The line should be steeper for the second leg than for the first leg of hare's journey. Both lines must be considerably steeper than the tortoise's line. Make sure the tortoise wins!)

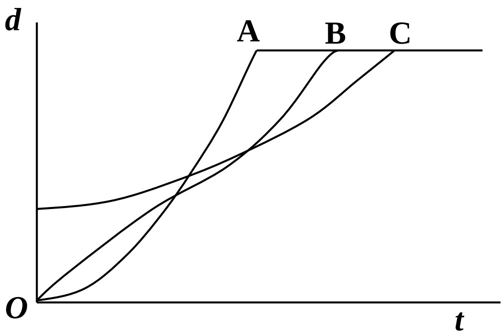


1.25.3 Draw a distance-time graph for your journey to school today. Put approximate values on the axes and annotate it to explain what's going on. You could use vertical lines to separate bus-walking-train, etc. regions.

If the journey to school makes a rather dull graph, pupils could choose a different journey – to a friend's house, on holiday, etc.

1.25.4 Draw on the board a graph of 3 people running a race (e.g., below). Pupils write a race commentary. Who is in the lead at the start? Who overtakes who in what order? Was it a close finish? Did people run at steady speeds or did they all gradually get quicker? Did any of them seem to run out of steam?

e.g.,



C runs slowest – perhaps that's why they gave him a head start. Anyway, he speeded up but still came last. A made a poor start but picked up speed and won. B started fastest but seemed to "take it easy", and A speeded up and overtook him as he slowed down. Once he saw that A was going to win he increased his speed but it was too late and he only came in 2nd.

Pupils might make these details into a more realistic "commentary"-style.

1.25.5 Draw a graph of a *Skoda* accelerating away from a standstill at traffic lights. On the same graph draw a *Ferrari*.

Could discuss "0 to 60" values as a measure of acceleration.

1.25.6 Draw a distance-time graph for a dog running round in a circle (chasing its tail).

What if it gets faster and faster?

What about a cat chasing a dog in a circle?

Answer: to draw a normal distance-time graph, you need to consider just 1 direction of motion. Therefore, at a steady speed you get a sine wave. Motion at an increasing speed gives a sine wave of increasing frequency (decreasing time period). The cat's motion would also be sinusoidal at a similar frequency, though out of phase.

1.25.7 Alex runs round and round a race track taking 40 seconds to complete each circuit. Beth runs the opposite way round the track and meets Alex every 15 seconds. How long does it take Beth to complete each circuit of the track? Assume that they both run at constant speed and never get tired.

If Clare runs in the same direction as Alex and passes him every 60 seconds, how often do Clare and Beth pass each other?

Answers: Could sketch distance-time graphs and use similar triangles.

Alternatively, Alex's speed = $\frac{1}{40}$ *circuit/sec.*
If Beth's speed = b *, then* $\frac{1}{40}+b=\frac{1}{15}$ *so* $b=\frac{1}{15}-\frac{1}{40}=\frac{1}{24}$*, so Beth takes 24 seconds to complete a circuit. If Clare's speed =* c *, then* $c-\frac{1}{40}=\frac{1}{60}$ *so* $c=\frac{1}{40}+\frac{1}{60}=\frac{1}{24}$*, so Clare is running at the same speed as Beth. So they meet each other twice during each circuit, so they pass every 12 seconds.*

1.25.8 Graphs of height, diameter, etc. versus volume or time as liquids are poured at a steady rate into oddly-shaped containers.
You can actually do it reasonably well (qualitatively)

It may be useful to mention **rate =** $\dfrac{\textbf{how much}}{\textbf{how long}}$*;*
e.g., litres/min or mph or £ per kg, etc.

with two jugs of water and some common containers (measuring cylinders, conical flask, beaker, test-tube from science dept.).

Teacher demonstration is the safest!

1.25.9 A caterpillar crawls at a speed of 40 cm per hour towards a bed of cabbages. After eating his fill, he crawls back (more slowly) at only 10 cm per hour. What was the caterpillar's average speed over the whole journey?
The answer *isn't* 25 cm/hour.

Answer: 16 cm per hour.
He spends four times as long returning as he took getting there (since he travels at only a quarter of the speed), so the mean speed is $\frac{1\times 40+4\times 10}{5}=\frac{80}{5}=16$ *cm/hour.*
(You could draw a distance-time graph to visualise this.)

1.25.10 A train leaves London at 9.00 am heading for Edinburgh and travelling at 100 mph. At the same time a train leaves Edinburgh travelling at 120 mph towards London. When they pass each other, which train is closer to London?

Answer: They must be the same distance from London when they pass, obviously!

1.25.11 A train leaves Birmingham New Street Station and accelerates at a steady rate up to a speed of 100 mph. It maintains this speed for 30 mins before decelerating at a steady rate to a stop.
If the whole journey takes 35 mins, how far has the train travelled?

Answer:
The easiest way of solving this is to draw a speed-time graph and calculate the area underneath it.
The graph is a trapezium of "height" 100 mph. The "top length" is 0.5 hours and the "bottom length" is the total journey time $\frac{35}{60}$ *hours.*
Therefore, the total distance = $\frac{1}{2}(a+b)h$
$=\frac{1}{2}(\frac{1}{2}+\frac{35}{60})100=54.2$ *miles.*

1.25.12 Pupils may have come across the *suvat* equations in Science, but may not realise that they relate to velocity-time graphs (see sheet).

These equations are valid only for constant (uniform) acceleration.

1.25.13 Zeno's Paradox of the Tortoise and Achilles (about 500 BC). The idea is that a tortoise challenges a man called Achilles to a race. Although the tortoise will obviously run slower than Achilles, the tortoise claims that he can still win so long as he gets a bit of a head start. Let's imagine that the race is 100 m and that Achilles can run at 10 m/s, whereas the tortoise can only do 1 m/s. Let's say that Achilles gives him a 10 m start. What will happen?

Here is the tortoise's (wrong) argument: One second after the race starts, Achilles will have run 10 m and is at the spot where the tortoise started, but he doesn't catch up with the tortoise yet because the tortoise has moved forward 1 m during that second. It takes Achilles another 0.1 second to catch up with where the tortoise is now, but by then the tortoise has moved on a further 10 cm. No matter how many times Achilles tries to catch him up, he'll always be a tiny bit behind – he'll never be able to overtake him.

This can make an interesting class or group discussion.

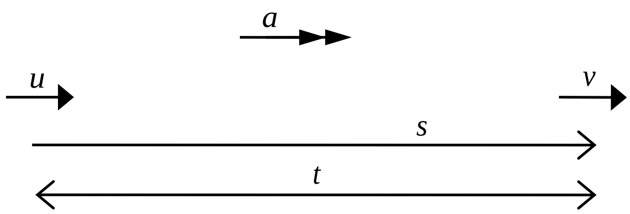
A paradox *is something that sounds right but doesn't seem to make sense.*

These aren't realistic values; they're just nice numbers to make the paradox easier to grasp.

Commonsense says that Achilles will take 10 seconds to get to the finish, whereas the tortoise will take 90 seconds (he has to run only 90 metres), so Achilles will still win by 80 seconds. This is in fact correct.

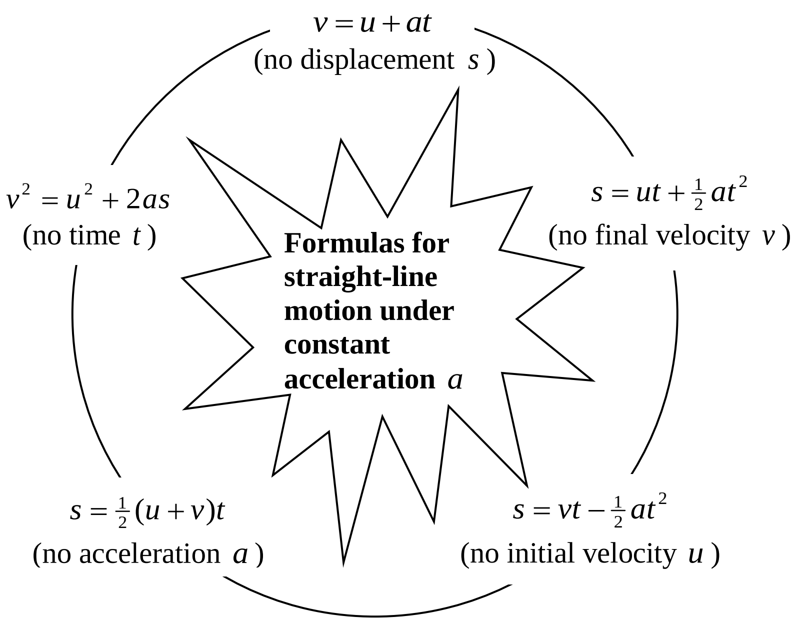
The tortoise is wrong because after 1.1111… seconds Achilles will catch him up and then overtake him – and after 10 seconds he'll have won the race. The mistake is to think that because the time 1.111… (recurring) seconds has infinitely many digits it is an infinite amount of time; it isn't, it's in between 1.1 and 1.2 seconds. ($10t=10+t \Rightarrow t=\frac{10}{9}$ *when they meet.)*

Displacement, Velocity, Acceleration



A particle initially has a velocity u and moves a displacement s under an acceleration a so that it ends up with a final velocity v at a time t later on.

There are five so-called *suvat* equations:



Different Units for Speed

mph

0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95

kph

0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150

m/s

0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42

	$\div 1.15$		$\times 1.6$		$\times \frac{5}{18}$	
	$\longrightarrow$		$\longrightarrow$		$\longrightarrow$	
knot		**mph**		**kph**		**m/s**
	$\longleftarrow$		$\longleftarrow$		$\longleftarrow$	
	$\times 1.15$		$\div 1.6$		$\div \frac{5}{18}$	