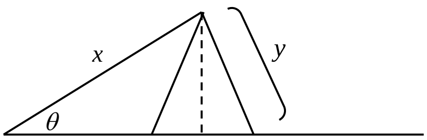
2.12 Similarity and Congruence

- These terms can apply to plane shapes or to solids: *similar* means that one shape/solid is an enlargement of the other; *congruent* means that the shapes/solids are identical (a special case of similarity with scale factor 1). Reflections (turning the shape over) count as congruent.
  Pupils may want to use the equals sign to indicate congruence (e.g., $\Delta ABC = \Delta PQR$), and this might be an opportunity to discuss whether = always means the same thing in maths or not.
- For material on similarity, see section 2.13.

2.12.1 Two triangles will be congruent if one of these conditions is true.

1. the three sides of one of the triangles are the same as the three sides of the other (SSS);
2. two sides and the angle in between are the same (SAS);
3. two angles and the side in between are the same (ASA);
4. the triangles are right-angled and the hypotenuse and one other side match (RHS).

Why are these the only conditions that guarantee congruence?



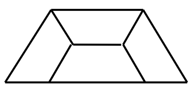
There is much logical thinking involved here.

If all three angles match (AAA) then the triangles must be similar and might be congruent but needn't be.

If two sides match (SS), the third side can be anything between the sum of the two given sides and their difference, so there are infinitely many possibilities.
One side and any two angles (SAA) is equivalent to ASA because in a triangle the angles must add up to 180°, so given two angles you can always work out what the third must be, but the angles and sides must correspond.

The crucial case to think through is ASS (the angle isn't *between the two given sides).*
Here, what happens depends on how long the second side is relative to the first.
If the angle is θ, the first side has length x and the second side has length y, then we get the diagram on the left.
If $y < x\sin\theta$, the sides don't join up and there is no such triangle.
If $y > x\sin\theta$, then there are two possible triangles depending on which side of the vertical dashed line the third side goes.
*If $y = x\sin\theta$, then there is just one possible triangle and it's right-*angled *(this is the RHS possibility mentioned already).*

2.12.2 Find some shapes that can be cut up into two or more pieces which are all mathematically similar to the original shape.
Start with triangles.



Answers: (number of similar shapes produced in brackets)

- *any right-angled isos. triangle (2, 3, 4, …);*
- *any equilateral triangle (4, 7, 9, 10, …);*
- *any parall'm, including rectangles, with sides in the ratio $1:\sqrt{2}$ (2) (like A-size paper);*
- *any parall'm at all, including rhombuses, rectangles and squares (4, 9, 16, …);*
- *special trapeziums, e.g., see left (4)*
- *lots more possibilities*