2.15 Dimensions and Units

- Although the practical everyday relevance is clear, this can be a dull topic unless there is some purpose to converting quantities from one unit to another. This topic works best by combining with others; e.g., standard form, volume/area, estimation, etc. There are some suggested problems below.
- It's hard to say exactly what "dimensions" are. You could ask pupils if they know what the "d" stands for in "3d" and then see what they think there are "3" of. The answer is something like "mutually perpendicular directions". Mathematicians often talk about 4 or more dimensions. In Maths, extra dimensions often don't make things that much harder to calculate, but it gets harder/impossible to visualise!

2.15.1 Conversion graphs.
Find out currency conversion rates from newspapers or the internet.

Bringing along foreign coins adds interest. See if pupils can identify the country and estimate how much the coin is worth in our money.

Pupils can draw, for example, value in French Francs against value in British Pounds on one graph, and German Marks against British Pounds on another. Pupils can then convert Francs to Marks using one graph after another (pick several values) – the resulting graph of Marks against Francs should also be a straight line through the origin.

It's much easier to be a millionaire in some countries than in others!

Could discuss stock-markets, inflation, etc.

Are conversion graphs always straight lines?

Not necessarily; e.g., dropping a ruler between someone's fingers to measure their reaction time – the graph to convert cm to seconds is a curve.

(Actually, time in seconds = $\sqrt{\dfrac{d}{490}}$ *, where* d *= distance fallen in cm.)*

(Each 1 cm fallen counts for less as time goes on, because the ruler is speeding up.)

Do they always go through the origin?

Again, not necessarily; e.g., °C to °F.

2.15.2 Which is bigger, an imperial ton or a metric tonne? Are they different in the UK and the US? (Could find out for homework.)

Answer: Imperial is spelt "ton"; metric is spelt "tonne".
A UK ton is 2240 lb (a so-called "long ton" or "gross ton"), whereas a US ton (a "short ton" or "net ton") is only 2000 lb.

Since a metric tonne is 1000 kg (anywhere!), and there are 2.205 lb in a kg, a metric tonne is 2205 lb, so this is in between (see left).

So the order is
UK ton > metric tonne > US ton.

What about gallons?
Similarly, US pints are less than UK ones, but US fluid ounces are more*, since in the US there are 16 fluid ounces in a pint, whereas in the UK there are 20!*

A UK gallon is 4.55 litres, whereas a US gallon is only 3.79 litres.

2.15.3 Estimate the total mass of everyone in the room?

Answer: Assume an average pupil weighs 50 kg. A class of 30 would weigh 30 × 50 kg = 1500 kg or 1.5 tonne.

What about the total mass of everyone in school assembly?

Depends on the size of the school, obviously.
(Be cautious if anyone might be sensitive about this task.)

2.15.4 Find out how high up aeroplanes typically fly.

How high are the tallest buildings?

Answers: (Note that 5280 ft = 1 mile.)
- ***aeroplanes**: e.g., 30 000 ft = 6 miles (approx) (The SR71 spy-plane flew at an altitude of 16 miles, but the pilots had to wear space-suits!);*

How high up are satellites?

How far away is the moon/the sun?

- ***tallest buildings**: (lots of debate over exactly what counts) around 500 m or nearly 2000 ft;*
- ***satellites**: anywhere from 100's of miles to tens of thousands of miles; e.g., geostationary satellites are at 22 223 miles (the further out they are the longer they last because there's less material for them to bump into);*
- ***moon** (a natural satellite): 240 000 miles;*
- ***sun** (a star): 93 000 000 miles.*

It's impossible to draw them all on a linear scale.

Can you draw a scale diagram to illustrate? (possible homework)

Find out how astronomers measure distances?

*The mean distance from the earth to the sun is called an **"astronomical unit" (AU)**, 1.5×10^{11} m, or 9.3×10^{7} miles. For example, astronomers might say that the distance of mercury from the sun is 0.39 AU, whereas for Pluto it is 40 AU.*

***"Light years" (ly)** are another way of measuring* distance *(not time); a light year is the distance light travels in a vacuum in one year and is 10^{16} m approx.*

*Astronomers also use **"parsecs" (pc)**, and 1 parsec = 3.26 ly = 3×10^{16} m.*

What about leagues and fathoms?

They're used in sea-travel.
1 fathom = 6 feet;
1 league = 3 miles
(1 nautical league = 3 nautical miles;
a nautical mile = 1.15 land miles.)

2.15.5 Estimate the number of tubes of toothpaste used per year in the UK.

What assumptions do you have to make?

Answer: Assume that there are 60 million people in the UK and that everyone brushes their teeth on average once a day (some more, some less). Assume all tubes hold 75 g toothpaste and that everyone uses 1 g for each brushing.
Therefore, for 365 days (leap years make no significant difference) we'll use
$365 \times 1 \times 60 \times 10^{6}$ g = 2×10^{10} g, which corresponds to $2 \times 10^{10} \div 75$ tubes = 3×10^{9},
3 billion tubes per year (approx).

2.15.6 How many pencils would it take to stretch across a football pitch from one goal to the other?
How many to stretch to the top of the Eiffel Tower?
How many to stretch a mile?
How many to go all the way round the world at the equator?
How many to go to the moon and back?

Answers:
Take an average pencil as 15 cm long.
***Football pitch** = 100 m long, so about 700.*
***Eiffel Tower** = 324 m high, so about 2000.*
***A mile** = 1600 m, so about 11 000 pencils.*
***Equator** = $2\pi r$ where r = radius of the earth = 6.4×10^{6} m, so equator = 4×10^{7} m so about 3×10^{8} pencils (300 million).*
*Average **distance to the moon** = 4×10^{8} m, so twice this is 8×10^{8} m, so about 5×10^{9} pencils (5 billion).*

2.15.7 Dimensions.
Tell me a kind of shape and how to work out its area.
We'll write it as a formula.
e.g., square: l^2 ; triangle: $\frac{1}{2}bh$; etc.
Now tell me some solids and how to work out their volumes.
What do you notice?

You can't do this topic until pupils are familiar with finding areas and volumes of a number of different shapes/solids.

Record in two columns on the board: "area formulas"; "volume formulas".

Area is always found by multiplying two lengths together (possibly also multiplying by a fixed number); Volume is always a length multiplied by a

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If we write this as L^2 and L^3, then this just means "some length" squared/cubed.

The formula for the area of an ellipse is one of these. Which one?

πabc ; πab ; $\pi a^2 b$; πab^2 ; $\pi(a+b)$

It's worth reinforcing the point that $A \Rightarrow B$ does not mean that $B \Rightarrow A$. Right formula $\Rightarrow$ right dimensions, but right dimensions $\not\Leftarrow$ right formula. $A \Rightarrow B$ does *mean that $B' \Rightarrow A'$ (where B' means "not-B"). So dimensions wrong $\Rightarrow$ formula definitely wrong.*

2.15.8 Check out the "dimensional soundness" of some Physics formulas; e.g.,

- Newton's 2nd Law: $F = ma$
  $[F] =$ Newtons
  $[ma] = MLT^{-2}$
  so 1 Newton is defined as 1 kg m/s^2;
- constant acceleration formulas; e.g., $v = u + at$ and $v^2 = u^2 + 2as$;
- work done and energy formulas; e.g., $W = Fs$ and $E = \frac{1}{2}mv^2$;
- the time period of a pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$;
- wave motion: $v = f\lambda$;
- lenses: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$;
- electricity; e.g., $V = IR$, $P = VI$, $Q = CV$;
- magnetism; e.g., $T = BANI\cos\alpha$;
- fields; e.g., $F = -\frac{Gm_1m_2}{r^2}$, $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$;
- pressure; e.g., $pV = nRT$, $p = \rho hg$;
- radioactivity; e.g., $N = N_0 e^{-\lambda t}$;

and many others.

length multiplied by length, or an area multiplied by a length (and possibly multiplied by a constant).

πab, since this is the only formula with L^2 dimensions. (a is half the length of the major axis (longest diameter) and b is half the length of the minor axis (shortest diameter).)

Using dimensions never helps us to get the constant right, for example.
So dimensions would never tell us to put in the π in the formula for the area of the ellipse.

In Mechanics, you also use M and T for mass and time. Other areas of Science require temperature θ, current A and even luminous intensity I. Most things can be made up from combinations of these, or else they're dimensionless (e.g., angles).

See Physics books for definitions of these quantities.

- *You can think of this formula as defining a Newton as the force necessary to accelerate a 1 kg mass by 1 m/s^2;*
- $[at] = LT^{-2}T = LT^{-1}$;
- $[Fs] = MLT^{-2}L = ML^2T^{-2}$ *= Joule;*
- $\left[\frac{1}{2}mv^2\right] = M(LT^{-1})^2 = ML^2T^{-2}$;
- $\left[2\pi\sqrt{\frac{l}{g}}\right] = \left(\frac{L}{LT^{-2}}\right)^{\frac{1}{2}} = \left(\frac{1}{T^{-2}}\right)^{\frac{1}{2}} = T$;
- $[f\lambda] = T^{-1}L = LT^{-1} = [v]$;
- $[u] = [v] = [f]$;
- $[V] = \frac{ML^2T^{-2}}{AT} = A \times ML^2T^{-3}A^{-2}$ *, etc.;*

Dimensions can help with remembering the units of constants such as $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1} and $h = 6.63 \times 10^{-34}$ J s.

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Metric and Imperial Measurements

	Length	Volume	Mass	
METRIC	**mm** $\underset{\times 10}{\overset{\div 10}{\rightleftarrows}}$ **cm** $\underset{\times 100}{\overset{\div 100}{\rightleftarrows}}$ **m** $\underset{\times 1000}{\overset{\div 1000}{\rightleftarrows}}$ **km**	**cm^3** = **ml** $\underset{\times 1000}{\overset{\div 1000}{\rightleftarrows}}$ **litre** $\underset{\times 1000}{\overset{\div 1000}{\rightleftarrows}}$ **m^3**	**mg** $\underset{\times 1000}{\overset{\div 1000}{\rightleftarrows}}$ **g** $\underset{\times 1000}{\overset{\div 1000}{\rightleftarrows}}$ **kg** $\underset{\times 1000}{\overset{\div 1000}{\rightleftarrows}}$ **tonne**	**METRIC**
CONVERSION	**cm** $\underset{\times 2.5}{\overset{\div 2.5}{\rightleftarrows}}$ **inch** **km** $\underset{\times 1.6}{\overset{\div 1.6}{\rightleftarrows}}$ **mile**	**pint** $\underset{\times 1.7}{\overset{\div 1.7}{\rightleftarrows}}$ **litre** $\underset{\times 4.5}{\overset{\div 4.5}{\rightleftarrows}}$ **gallon**	**g** $\underset{\times 28}{\overset{\div 28}{\rightleftarrows}}$ **oz** **lb** $\underset{\times 2.2}{\overset{\div 2.2}{\rightleftarrows}}$ **kg**	**CONVERSION**
IMPERIAL	**inch** $\underset{\times 12}{\overset{\div 12}{\rightleftarrows}}$ **foot** $\underset{\times 3}{\overset{\div 3}{\rightleftarrows}}$ **yard** $\underset{\times 1760}{\overset{\div 1760}{\rightleftarrows}}$ **mile**	**floz** $\underset{\times 20}{\overset{\div 20}{\rightleftarrows}}$ **pint** $\underset{\times 8}{\overset{\div 8}{\rightleftarrows}}$ **gallon**	**oz** $\underset{\times 16}{\overset{\div 16}{\rightleftarrows}}$ **lb** $\underset{\times 14}{\overset{\div 14}{\rightleftarrows}}$ **stone** $\underset{\times 160}{\overset{\div 160}{\rightleftarrows}}$ **ton**	**IMPERIAL**

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