2.16 Compound Measures and Rates of Change

- To use time in calculations (e.g., working out speed) pupils need to convert hours and minutes either to decimal hours or to minutes (see the first task below).
- Don't assume all pupils will be confident reading an analogue clock.
- See section 1.25 for further ideas.

2.16.1 Decimal Time.
If I went on a journey and said it took me 3.25 hours, why might that be confusing?
How long do I really mean?

Answer: Do I mean 3 hours and 25 minutes or do I mean $3\frac{1}{4}$ hours?
3.25 h = 3h15min

How long is 3.7 hours literally?

Less than 3h45min (3.75 h, $3\frac{3}{4}$ h).
More than 3h30min (3.5 h, $3\frac{1}{2}$ h).

Decimal time × 60 = time in minutes.

Could say that 0.1 h = 6 min, so 0.7 h = 42 min, so the time is 3h42min.

2.16.2 **NEED** local train/bus timetables (companies will sometimes give you as many as you want at no cost, especially if they're nearly out of date).

You may need to explain how the timetables work; i.e., different sides for different directions; "slow" and "fast" trains; different services Saturday/Sunday, etc.

Oral and mental work based on times.
e.g., "I want to get to London by 6 pm. Which train should I catch and how long will the journey take?"

Pupils need to apply commonsense bearing in mind that services may be delayed or cancelled.

2.16.3 What does it mean if an aeroplane travels at "mach 2.5"?

Answer:
The "mach" number (named after Ernst Mach, 1838-1916) is the number of times the speed of sound that the aeroplane is travelling.
Mach >1 means "supersonic".

The speed of sound in air is 330 m/s = 760 mph at sea level, but it drops considerably with altitude (e.g., it's only 590 mph at 30 000 ft) because of the decrease in density.

(You have to say the speed of sound in air *because sound waves need something to go through – the speed of sound in a vacuum is zero.)*

2.16.4 Do you think there's a limit to how fast any object can go?

Of course ordinary objects (e.g., an aeroplane) would fall to bits if we tried to make them go too fast, but Einstein's theory is more fundamental than that.

Answer:
According to Einstein's (1879-1955) relativity theory, no object can go faster than the speed of light in a vacuum (c).
$c = 3 \times 10^8$ m/s or 186 000 miles/s.
Sometimes other speeds are given relative to c; e.g., speed of electrons in a particle accelerator could be 0.9c.

2.16.5 When is speed measured in knots?

Answer:
It's a unit of speed often used for aircraft and boats;
1 knot = 1 nautical mile per hour
= 1.15 land miles per hour.

2.16.6 "Around the World in 80 Days", Jules Verne. What was Phileas Fogg's average speed?

Answer: $\frac{4\times10^4}{80\times24} = 21$ *kph (approx).*

2.16.7 Would you say we're moving at the moment?
The earth is rotating. Estimate how fast you think we're moving (mph).
What would you need to know to work out our

Answer:
On the equator, we move $2\pi r$ metres every 24 hours, which is $2\pi\times6.4\times10^6 = 4\times10^7$ m, corresponding to a speed of about 1700 kph or 1000

© Colin Foster, 2003 www.foster77.co.uk

speed?
Radius of earth = 6.4×10^6 m
(Hint: Imagine we're on the equator.)

Why doesn't it feel like it?

Of course, the earth is also orbiting the sun.

mph. Off the equator it's slower.

The angular speed is very low (0.25° per min), so we don't notice our direction changing. We can't tell the high speed because the atmosphere, etc. moves with us (like being on a very smooth train at night).

2.16.8 It takes 8 hours to fly from London to New York, a distance of 3 500 miles. What is the average aeroplane speed?
Concorde gets there in about $3\frac{1}{2}$ hours. What is Concorde's average speed?

If Concorde could fly non-stop around the world, how long would it take?

Answers:

440 mph (sub-sonic)

1000 mph (supersonic; Concorde cruises at about Mach 2)

$\frac{4\times10^4 \div 1.6}{1000} = 25$ *hours (just over a day).*

2.16.9 I am standing on the platform at a railway station. An inter-city train speeds through the station and it takes 4 seconds to pass me. A few moments later, another train of the same length comes through going the other way. This second train takes 5 seconds to pass me. How long did it take them to pass each other?

Answer: If x is the (unknown) length of the trains, then the speed of the first train is $\frac{x}{4}$ and the speed of the second is $\frac{x}{5}$. Their speed relative to each other will therefore be $\frac{x}{4}+\frac{x}{5}=\frac{9x}{20}$. When they pass each other there is a relative distance of $2x$ to cover, so the time taken will be $2x \div \frac{9x}{20} = \frac{40}{9} = 4\frac{4}{9}$ seconds.

2.16.10 Alison and Beckie run a 100 m race. Alison wins by exactly 1 m. If they run again, but this time Alison starts 1 m behind the starting line, who will win this time?
Assume that they both run at steady speeds and perform just as well on the second race.

Answer: Alison again. When Alison runs her first 100 m, Beckie will have got to 99 m, so they'll be level. But then in the next 1 m Alison will overtake and win by 1 cm.

2.16.11 If sound travels at 330 m/s, make up an easy to remember rule (or check one you already know) to work out how far away lightning is when you see the flash and hear the thunder.

Answer: The speed is roughly 1 km every 3 seconds, so one possibility would be "count the seconds from the flash to the thunder – could say 'zero' on the flash – and divide by 3 to find out the distance away in km".

2.16.12 Density. Which weighs more, 1 kg of wood or 1 kg of steel?
What is different about 1 kg of wood and 1 kg of steel?

Answer: the same, of course!
The steel would take up much less space (volume) than the wood would.

Work out the mass of a cuboid gold bar that is 18 cm by 9 cm by 4.5 cm.
The density of gold is 19.32 g/cm^3.
Do you think you could lift one?

Volume = 18 × 9 × 4.5 = 729 cm^3.
So mass = 729 × 19.32 = 14 kg (or about 30 lb).

Yes. About 14 bags of sugar, or half a sack of potatoes!

Could work out how much it would be worth. Prices of gold vary minute by minute, but they're in the region of £7 000 per kg.

This would give a value of about £100 000.

What would be the value of a silver bar the same size? (The density of silver is 10.49 g/cm^3; the cost is roughly £100 per kg.))

You could work it out as above, or scale down.

cost $= 100000 \times \frac{10.49}{19.32} \times \frac{100}{7000}$ *= £ 800 approx.*

2.16.13 How dense are we?!
Average human volume is about 70 litres (see section 2.10.14) and average human mass is around 70 kg, so average human density is about 1 kg/litre or 1 g/cm^3. This is the density of water, and that explains why we float, but only just.

This is "average" human density in two senses. Not all human beings are identical, of course, but also the body is non-uniform. Bones are dense and sink, whereas lungs are relatively light. So this is average density over the whole body as well as the whole population.

© Colin Foster, 2003 www.foster77.co.uk

Speed Measured in Different Units

mph

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95

kph

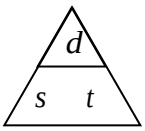
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150

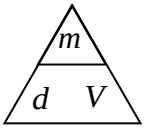
m/s

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42

	÷1.15		×1.6		$\times \frac{5}{18}$	
	→		→		→	
knot		**mph**		**kph**		**m/s**
	←		←		←	
	×1.15		÷1.6		$\div \frac{5}{18}$	

The triangle on the left gives the formulas for speed s, distance d and time t.
The one on the right gives the formulas for density d, mass m and volume V.





Densities of Common Materials

material	density (g/cm^3)	mass of 50 cm^3	mass of 35 cm^3	volume of 50 g	volume of 35 g
water	1.00	50.0	35.0	50.00	35.00
aluminium	2.70	135.0	94.5	18.52	12.96
zinc	7.13	356.5	249.6	7.01	4.91
iron	7.87	393.5	275.5	6.35	4.45
copper	8.96	448.0	313.6	5.58	3.91
silver	10.49	524.5	367.2	4.77	3.34
lead	11.36	568.0	397.6	4.40	3.08
mercury	13.55	677.5	474.3	3.69	2.58
gold	19.32	966.0	676.2	2.59	1.81

© Colin Foster, 2003 www.foster77.co.uk