3.5 Probability

- A topic where there are many misconceptions, partly due to problems with ratio thinking, partly due to not recognising the assumptions behind the theory. A crucial concept is the one of "equally likely outcomes". Pupils can mindlessly put one number over another number to create fractions that represent probabilities without any understanding of the size of these values. For this reason it can be an advantage to use decimals some of the time in early probability work.
- You could allow pupils to leave fractions unsimplified if that is a major hurdle in this topic.
- Sample space diagrams are generally easier to draw and use than tree diagrams, but are limited to situations where there are two events.
- For three or more events, tree diagrams are needed. Some "blanks" are given on sheets for those pupils who find it hard to judge the layout: when drawing tree diagrams, it can sometimes be easiest to start at the right side (where there are lots of branches) and work your way to the left.
- You may need to teach the composition of a standard pack of cards:
  Excluding jokers, there are 52 cards made up of 4 suits (hearts, diamonds, spades and clubs) each containing 13 cards (ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king). Hearts and diamonds are red; spades and clubs black. Ace may be "high" or "low". Jack, queen, king count as "picture cards". (Some pupils/parents may object to gambling contexts or any use of playing cards.)
- For practical work, throwing dice is much quieter if they're thrown onto a book or into a small cardboard box rather than onto a desk! – a box also makes them less likely to fall on the floor.
- Dictionaries now seem to allow *dice* (as well as *die*) as the singular.

3.5.1	Probability Scales. Everybody talks about probability using words such as "likely", "even chance", "almost certain", etc. Mathematicians simply make it more precise by using a scale of numbers. Numbers between 0 and 1 can be fractions, decimals or percentages.	*It's like the difference between describing temperature as "fairly hot" and using the Celsius temperature scale.* *(Except that the probability scale has ends at 0 and 1. The temperature scale has an end at absolute zero,–273 °C, but no clear cut-off at the upper end.)*
3.5.2	**NEED** carrier bag and coloured cubes. A useful way of discussing simple probability. If I want a probability of 0.5 (even chance) of pulling out a red cube, what could I put in?, etc.	*You need to use a thick or dark-coloured carrier bag so that you can't see the cubes inside. (Cloth money bags, available from banks, are ideal for this.)*
3.5.3	Imagine I have 12 red cubes and 4 blue cubes in a bag. So p(red) = 0.75 and p(blue) = 0.25. Imagine I will give you 20 pence if you correctly guess the colour of the cube I pull out at random. Imagine we play this game again and again. What would you guess?	*Answer:* *The best strategy is to guess* red *every time, because if the draws are truly random then every go is more likely to turn out red than blue.* *Some pupils will feel that you ought to guess blue a quarter of the time, but the problem is that we don't know which quarter of the goes will be blue! This gets us to the idea of "independent" events.*
	How much would be a fair amount to charge people to play this game?	*Clearly 20p or more would be too much (no-one would play!). On average, if people guess "red" every time, they will win 0.75 × 20 = 15 p per go, so in the long run charging 15 p you would break even. Perhaps you would charge 15 p and hope that people will not always choose red, and so on average you should make a profit.*
3.5.4	**NEED** "Number Probabilities" sheet. This is an opportunity to review words and concepts like multiple, factor, prime, square, triangle, power, etc.	*Answers:* *1.* $\frac{1}{10}$ *; 2.* $\frac{1}{5}$ *; 3.* $\frac{1}{2}$ *; 4.* $\frac{1}{10}$ *; 5.* $\frac{1}{25}$ *; 6.* $\frac{19}{20}$ *; 7.* $\frac{1}{11}$ *;* *8.* $\frac{9}{10}$ *; 9.* $\frac{1}{100}$ *; 10.* $\frac{3}{50}$ *(remembering that 1 is a factor of every number); 11.* $\frac{2}{25}$ *; 12.* $\frac{1}{50}$ *; 13.* $\frac{9}{100}$ *;*

If you can afford to use the sheets only once, pupils may prefer to colour in the appropriate numbers in the number square for each question as they go.
Or you could use the 100-squares given in section 1.16.

14. $\frac{1}{10}$ (remembering that 1 is the first square number); 15. $\frac{1}{2}$; 16. $\frac{7}{100}$; 17. $\frac{3}{50}$ (or $\frac{7}{100}$ if you count $2^0=1$); 18. $\frac{33}{100}$ (not $\frac{1}{3}$); 19. $\frac{13}{100}$; 20. $\frac{1}{4}$ (remembering that 1 doesn't count as a prime number).

3.5.5 Invent a Fair Game.
What about this one – is it fair?
Who's likely to do better out of it in the long run?
"You pay me £1 to play. You throw two fair dice and if you get a double I give you £5. Otherwise you lose your money."

Pupils need to be encouraged to keep it very simple. Even relatively simple games can be highly challenging to analyse!

Not fair on you, because p(double) = $\frac{1}{6}$, so

	your gain per go
1. You pay me £1.	-1
2. Throw the dice.	$+5\times\frac{1}{6}$
total	$-1+\frac{5}{6}=-\frac{1}{6}$

So on average I will gain £1 of yours every 6 goes.

Can you alter this game to make it fair?

We could add this rule: "If you get a total score of 7, I have to give you your money back."
Since there is a $\frac{1}{6}$ chance of this happening, overall neither of us would make or lose money playing the game.

These games seem pretty pointless, since there is no element of skill, but games of chance are popular, perhaps partly because people don't understand probability! However, some people enjoy the game even though they don't expect to win anything playing.

3.5.6 **NEED** dice. Two Dice Experiment.
A fun way to do this is to have a "horse race".
On scrap paper, draw out a table as on the right (or use the sheet).

The 12 rows represent the tracks for 12 horses.
Choose a horse to "bet" on (no money!).
Then throw two dice, add up the scores and that horse moves on one place.
Keep going until a horse reaches the finishing line. That one's the winner.

This works best in pairs. Each person bets on a different horse. One person throws the dice and the other marks with an X the current positions of the horses.

After seeing what happens, try to make a more successful bet on your second go.

Is this a fair game? Are some horses more likely to win? Why is that?

Many pupils are shocked to discover that the scores with two dice aren't all equally likely.

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(ten columns after the column of numbers)

Some pupils may realise quickly that horse 1 is a no-hoper!

Sample Space diagrams are the ideal way to explain the results.

3.5.7 **NEED** "Statements About Probability" sheets.
One way to use these is to cut them into 10 separate statements and distribute them to different people.
Then hold a discussion in which one by one people read out their statements and they or others comment on them.
"Do you understand what they're meaning?"
"Why would someone think that?"
"Do you agree?"

Answers:
1. *Not true. They're not equally likely events; e.g., consider snow/not, earthquake/not, winning/not winning the National Lottery!*
2. *Not true. Proportionate thinking is necessary, and the probability of getting red from bag A ($\frac{2}{3}$) is actually greater than from bag B ($\frac{10}{17}$).*
3. *The coin is likely to be biased, but it's*

"Can you explain why in your own words?"
"Can you think of another example of that?"

Alternatively, pupils can discuss the statements in small groups.

Many pupils will be quite happy with the statements as they are, so this is a good way to tackle misconceptions.
You may not manage to convince everybody about everything in this one task!

theoretically possible for an unbiased coin to do this.

4. *Not true. The probability is* $\left(\frac{1}{2}\right)^{50}$ *in both cases, because each throw is an inde-pendent event: the coin has no memory!*
5. *If the outcome is affected by skill or "luck" then the probability is not exactly* $\frac{1}{6}$*. This value refers to a random process.*
6. *Similar to 5. Selecting different cards are not equally likely outcomes for this person.*
7. *Similar to 4. Not true. The probability is* $\left(\frac{1}{6}\right)^{2}$ *in both cases, because each dice is unaffected by the other: independent events.*
8. *Not true. Two consecutive tickets are no less likely than any other pair. If the draw is fair, every ticket is equally likely to be picked. So are every pair of tickets.*
9. *You could argue about the biology, but births are probably independent events, and the probability of a baby boy is about* $\frac{1}{2}$ *(but in fact just slightly greater). This misconception is often called the "Gambler's Fallacy".*
10. *Although it's silly, it can be a tricky one to explain. Whether someone else decides to bring a bomb is not affected by whether I do, because they're independent events.*

If the probability of the birth of a boy is greater than the probability of the birth of a girl, why isn't the world over-populated with men?
A complicated issue, but part of the answer is that baby girls are slightly more likely to survive into adulthood.

3.5.8 If you throw two ordinary dice, you can get any number from 2 to 12, but they're not all equally likely. By changing the numbers on the faces of the dice (say, using stickers), can you create two dice so that every score from 2 to 12 is equally likely?

Yes, but not simply – you have to "waste" some throws. One method would be to say that you will throw again if you get (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5), (4,2), (4,3), (4,4), (4,5), (5,2), (5,3) or (55). This leaves 22 possible combinations (which is divisible by 11), so now we can count the total score on the dice, except that (2,2) has to count as 2 (not 4) and (5,5) has to count as 12 (not 10). So long as you remember that, it isn't too bad, but it's pretty complicated.

You could use 33 of the 36 combinations but it's harder to make a close link between the numbers on the dice and what you need to count as the score.

Can you do it if you want every total from 1 to 12 to be equally likely?

Yes (much easier); e.g., change one dice so that three faces show a 0 and the other three a 6.

3.5.9 If you wanted a series of random numbers between 1 and 5, what kind of dice would you need?

A normal 6-sided dice would do, if you just ignore the throw every time you get a 6 (or stick a piece of paper over the 6 – although that might affect how the dice lands).

What if you wanted random numbers between 1 and 8?

You'd need a dice with more faces or a more complicated system.

3.5.10 Three Dice Experiment.
Choose a number between 1 and 20 – you can't change it once you've chosen.
Throw three dice 20 times and do a tally of the total scores.
Which score comes up most often?
Change your target number. What is the best target number?

This is an interesting, and much harder, variation for pupils who are already familiar with the "Two Dice Experiment".

(1, 2, 19 and 20 are impossible scores.)

In fact, the most probable score is 10 or 11.
You can pool results from the whole class.

Can we explain the results?
How can you get a total of 5, say?

1, 1, 3 (3 ways) or 1, 2, 2 (3 ways), so 6 ways in total. (You can imagine that the three dice are red, blue and green if that helps to see the number of permutations of each combination.)

Divide the possible totals among the class so that in groups pupils work out the number of ways of getting each total.

Pool the results and put them into a table (see sheet).

(There is one way of arranging AAA, 3 ways of arranging AAB and 6 ways of arranging ABC.)

3.5.11 Buffon's Needle (Comte de Buffon, 1707-1788). This 18th century problem asks what the probability is of dropping a needle onto a set of parallel lines and it landing *across* one of the lines. The length of the needle is the same as the spacing between the lines.

If the spacing x between the lines is larger than the needle length l, the probability of a hit becomes $\frac{2l}{\pi x}$.

Answer:
If the length of the needle is 1 unit and it falls at an angle θ to the lines, then it will cross the lines if its centre is closer than $\frac{1}{2}\sin\theta$ to the lines, and the locus of these points makes up a fraction $\sin\theta$ of the whole area, so the probability of a hit =
$$\frac{1}{\left(\frac{\pi}{2}\right)}\int_0^{\frac{\pi}{2}}\sin\theta\,\mathrm{d}\theta = \frac{2}{\pi}.$$
In principle you could use this method to estimate the value of π, but in practice it is a lot of work to get a reasonable degree of accuracy.
Computer simulations of the experiment do give an accurate value for lots of throws.

3.5.12 Paving Slabs (the Buffon-Laplace Needle Problem). **NEED** 2.5 cm long sticks and 5 cm × 5 cm squared paper (see sheet).

Pupils drop the stick onto the sheet of squares and record a "hit" if any part of the stick lies over any line, and a "miss" otherwise.

You can repeat the experiment (or different groups of pupils can try it) using shorter (1 cm) sticks.

In general, if l is the stick length and the grid rectangles are x by y ($l < x$ and $l < y$), then the probability of a hit is given by $\frac{2l(x+y)-l^2}{\pi xy}$. If we let $y \to \infty$, this probability becomes $\frac{2l}{\pi x}$, the result for parallel lines.

This is an alternative, based on Buffon's Needle (above), to the classic experiment where pupils drop a drawing pin and record whether it lands point up or point down.

You can plot a graph of "relative frequency so far" against the number of throws. This usually shows that the relative frequency settles down to a steady value – this value would be our estimate for the probability of a hit.

Using trigonometry and calculus, the theoretical probability of a hit is $\frac{7}{4\pi}$ = 0.56 (2 dp).
With a 1 cm long stick (probably the smallest practicable length), the theoretical probability of a hit is $\frac{19}{25\pi}$ = 0.24 (2 dp).

3.5.13 Working Out Probabilities (see sheet).

It's helpful to be explicit about the different methods we use to arrive at probabilities. Some pupils will think that "calculating is good and experimenting is bad".

3.5.14 Is *anything* absolutely impossible ($p = 0$) or completely certain ($p = 1$)?

Logically, if you are certain that something is impossible, then you have a complete certainty and an absolute impossibility, so you've killed two birds with one stone!

Answer:
This is really a philosophical question. Jokes aside (e.g., the probability of a certain football team ever winning a match, etc.), you could say that some things are intrinsically impossible; e.g., the probability of finding a triangle with four sides or discovering that 2 + 2 = 5.
Other non-mathematical contradictory events could be "me being at school and not being at school at the same time", etc.
As for complete certainty, some may argue that we may be completely sure about some religious statements because God is completely trustworthy.

3.5.15 How many people must you have together in a room before there is a greater than 50% chance of at least two of them having the same birthday?

Answer:
23 people, because p(everyone has a different

(We ignore birthdays on 29 February here.)

If there were more than 365 people, you wouldn't need to do any calculations, because the pigeonhole principle would say that there must be at least two people with the same birthday. (You can't share around 365 different birthdays among more than 365 people.)

birthday) = $\frac{{}^{365}C_{23}}{365^{23}} = 0.493$ *(3 dp), since there are only 365 different birthdays available. Since this is less than 0.5, and if you work it out for 22 people it's just greater than 0.5, then 23 is the minimum number of people.*

If you tried the above approach with more than 365, you would be trying to calculate ${}^{365}C_r$ *where* $r > 365$*, and that is meaningless.*

3.5.16 Find out what the "Monty Hall Problem" is – or tell them …

It's a famous conundrum. Imagine I have three identical-looking boxes, one of which contains a chocolate bar. I know which box the chocolate is in, but you don't.
You have to choose a box. Then I will open one of the *other two* boxes, but I will always open one with nothing in it. (I can always do that because there are three boxes, and even if you choose an empty one there will always be another empty one I can open.)
Now you can choose to stick with the box you've got or to swap to the other unopened box. The question is what is the best thing to do? Should you stay or swap?

In fact you're better off swapping. The easiest way to see this (and it's still not that easy to see!) is to imagine a hundred boxes instead of just three. After you've chosen a box, I open all the others except one (and except yours) and they're all empty. It's highly likely that the only reason I didn't open yours is that you'd chosen it. The chocolate is almost certainly in the other box that I declined to open. So swapping is definitely better.

An enjoyable discussion. If no-one has anything to say, give pupils time to discuss in pairs/groups before presenting their ideas to the whole class.

(Obviously, this assumes that you like chocolate and want to win the bar!)

This can lead to lively debate. It works well if the teacher is devil's advocate.
If they say "swap", you could ask "Has opening the third box really told you anything about the two unopened boxes?"
If they say "stay", you could ask "You picked it out of three possibles; now there are only two possibles isn't it more likely to be the other one?"

Another way to see it is like this: Call the boxes A, B and C. Imagine that the prize is in box A, but of course you don't know that. You must start by choosing A, B or C at random.
If you choose A, I can open B or C, but either way you lose if you swap.
If you choose B, I will have to open C and you will win if you swap.
If you choose C, I will have to open B and you will win if you swap.
So $\frac{2}{3}$ *of the time, swapping is better.*

3.5.17 A stick is broken into 3 pieces. What is the probability that they will make a triangle?

Answer: It turns out to be $\frac{1}{4}$*. You can prove it by thinking about 4 congruent equilateral triangles with sides equal to the length of the stick.*

Number Probabilities

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Use the number square above to help you work out these probabilities.
Write your answers as fractions in their simplest forms.

A number is chosen at random between 1 and 100 (inclusive).
Work out the probability that the number is …

1 a multiple of 10

2 a multiple of 5

3 a multiple of 2

4 greater than 90

5 less than 5

6 greater than 5

7 between 20 and 30 (inclusive)

8 a two-digit number

9 a three-digit number

10 a factor of 50

11 a factor of 24

12 a factor of 11

13 a multiple of 11

14 a square number

15 an odd number

16 a multiple of 13

17 a power of 2

18 a multiple of 3

19 a triangle number

20 a prime number

Statements about Probability

1 Tomorrow either it will rain or it won't rain.
Therefore the probability of rain tomorrow is 50%.

2 Bag A contains 4 red counters and 2 blue counters.
Bag B contains 10 red counters and 7 blue counters.
If you pick a counter out of bag B, you are more likely to get a red counter than with bag A because there are more red counters in bag B.

3 If you throw a coin 50 times and you get 48 heads and only 2 tails the coin must be biased.

4 If you throw a coin 50 times you are more likely to get 26 heads and 24 tails than to get exactly 25 of each, because getting exactly 25 heads and 25 tails is pretty unlikely.

5 Everyone knows that some people are luckier than others.
And everyone has good days and bad days.
So the probability of getting a 6 with a dice depends on who throws it and when.
You can't just say it's always $\frac{1}{6}$.

6 I know someone who's practised with cards and can pull out an ace whenever he wants to.
So for him the probability of getting an ace is nothing to do with chance.

7 If you throw two dice the chance of getting a double 6 is less than the chance of getting a double 5 because a double 6 would be the maximum score possible and there's only one way of that happening.

8 If you buy two raffle tickets from different places in the book, you are more likely to win than if you buy two consecutive tickets.
Consecutive tickets would be very unlikely to come up.

9 Next-door is a family with 4 boys.
By the law of averages, the next child is bound to be a girl.

10 The probability of there being a bomb on an aeroplane is 1 in a million.
The probability of there being ***two*** bombs on an aeroplane is much smaller.
Therefore if I take a bomb with me I'm much less likely to get blown up by someone else's bomb!

Two Dice Horse Race

FINISH↓

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Probability Summary

- **Event:** An **event** A, B, etc. is a possible outcome or a set of possible outcomes.
  e.g. A = throwing a fair dice and getting a 4,
  or B = throwing a fair dice and getting an even number.
  $n(A)$ is the number of equally likely ways of A happening.
  $n(\varepsilon)$ is the total number of equally likely outcomes (all the possibilities).
  $p(A)$ is the probability of A happening, so
  $$p(A) = \frac{n(A)}{n(\varepsilon)},$$
  and $p(A)$ must be between 0 (impossible, if there's no way of A happening) and 1 (certain, if A is the only thing that could happen).

- **The Complement of A:** This is the event "A doesn't happen".
  We use the symbol A'. E.g., if A = throwing a fair dice and getting a 4,
  then A' = throwing a fair dice and *not* getting a 4.
  It's always true that $p(A') = 1 - p(A)$.
  Sometimes if you want $p(A)$ it's easier to calculate $p(A')$ and use this formula to get $p(A)$.
  E.g., if A = throwing two fair dice and getting a total of at least 3, then
  A' = throwing two fair dice and getting a total of 2, so $p(A) = 1 - \frac{1}{36} = \frac{35}{36}$.

- **Combined Events:** If you have two events A and B, then
  $$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$
  $p(A \cup B)$ means the probability of *either* A *or* B happening (called "***A* union *B***")
  $p(A \cap B)$ means the probability of *both* A *and* B happening ("***A* intersection *B***")
  $p(A)$ includes the chance of A and B both happening [$p(A \cap B)$].
  $p(B)$ also includes the chance of A and B both happening [$p(A \cap B)$],
  so $p(A) + p(B)$ includes $2 \times p(A \cap B)$, and that's why we have to subtract $p(A \cap B)$ in the formula above (so we don't count it twice).
  You can see all of this more clearly on a **Venn diagram**.

- **Mutually Exclusive Events:** If A and B *cannot both* happen together (one excludes the possibility of the other), they're called **mutually exclusive** events.
  In that case $p(A \cap B) = 0$, so the formula above simplifies to
  $$p(A \cup B) = p(A) + p(B).$$
  But remember you can add probabilities like this only if they're mutually exclusive.

- **Conditional Probability:** We write $p(B|A)$ for the probability that B happens *given that* A *has already happened*. This is what you see on the 2nd branch of a **tree diagram**.
  You can only get there by going along the 1st branch, so A must already have happened.
  The probability that A happens and then B happens is
  $$p(A \cap B) = p(A) \times p(B|A).$$
  You can also write this formula as $p(B|A) = \dfrac{p(A \cap B)}{p(A)}$.

- **Independent Events:** If B is independent of A then it doesn't make any difference whether A happens or A' happens. In that case,
  $$p(B|A) = p(B|A') = p(B)$$
  Therefore $p(A \cap B) = p(A) \times p(B)$, but you must remember to use this only when events A and B are independent.

Sample Space Diagrams

A Total of Numbers on Two 6-Sided Dice

Let X = total score

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

X	$p(X)$
1	0
2	$\frac{1}{36}$
3	$\frac{1}{18}$
4	$\frac{1}{12}$
5	$\frac{1}{9}$
6	$\frac{5}{36}$
7	$\frac{1}{6}$
8	$\frac{5}{36}$
9	$\frac{1}{9}$
10	$\frac{1}{12}$
11	$\frac{1}{18}$
12	$\frac{1}{36}$
total	1

B Difference of Numbers on Two 6-Sided Dice

Let X = difference of scores

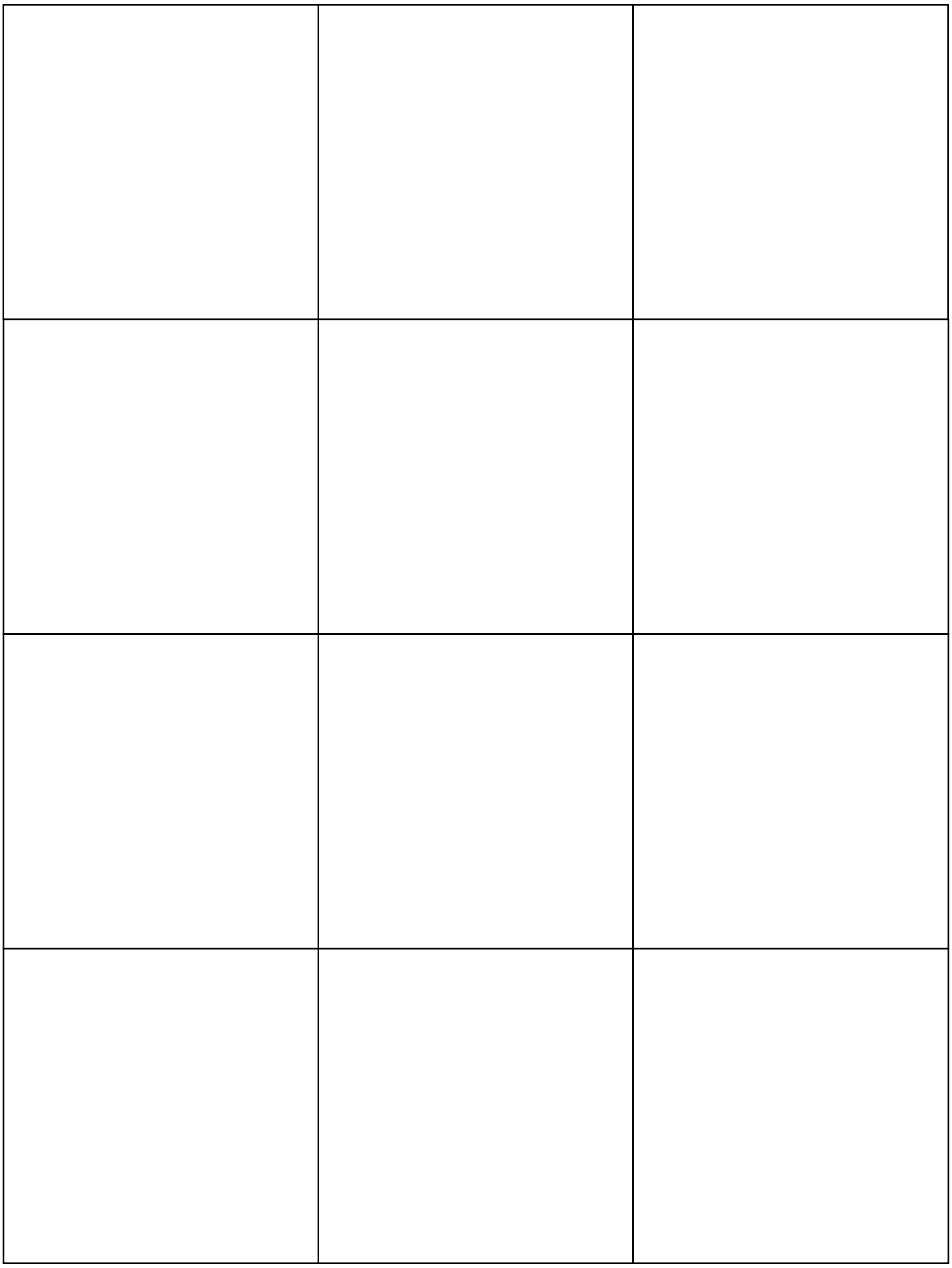
	1	**2**	**3**	**4**	**5**	**6**
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

X	$p(X)$
0	$\frac{1}{6}$
1	$\frac{5}{18}$
2	$\frac{2}{9}$
3	$\frac{1}{6}$
4	$\frac{1}{9}$
5	$\frac{1}{18}$
total	1

Three Dice Experiment

total	combination	no. of ways	total no. of ways	probability
3	1, 1, 1	1	1	$\frac{1}{216}$
4	1, 1, 2	3	3	$\frac{1}{72}$
5	1, 1, 3 1, 2, 2	3 3	6	$\frac{1}{36}$
6	1, 1, 4 1, 2, 3 2, 2, 2	3 6 1	10	$\frac{5}{108}$
7	1, 1, 5 1, 2, 4 1, 3, 3 2, 2, 3	3 6 3 3	15	$\frac{5}{72}$
8	1, 1, 6 1, 2, 5 1, 3, 4 2, 2, 4 2, 3, 3	3 6 6 3 3	21	$\frac{7}{72}$
9	1, 2, 6 1, 3, 5 1, 4, 4 2, 2, 5 2, 3, 4 3, 3, 3	6 6 3 3 6 1	25	$\frac{25}{216}$
10	1, 3, 6 1, 4, 5 2, 2, 6 2, 3, 5 2, 4, 4 3, 3, 4	6 6 3 6 3 3	27	$\frac{1}{8}$
11	1, 4, 6 1, 5, 5 2, 3, 6 2, 4, 5 3, 3, 5 3, 4, 4	6 3 6 6 3 3	27	$\frac{1}{8}$
12	1, 5, 6 2, 4, 6 2, 5, 5 3, 3, 6 3, 4, 5 4, 4, 4	6 6 3 3 6 1	25	$\frac{25}{216}$
13	1, 6, 6 2, 5, 6 3, 4, 6 3, 5, 5 4, 4, 5	3 6 6 3 3	21	$\frac{7}{72}$
14	2, 6, 6 3, 5, 6 4, 4, 6 4, 5, 5	3 6 3 3	15	$\frac{5}{72}$
15	3, 6, 6 4, 5, 6 5, 5, 5	3 6 1	10	$\frac{5}{108}$
16	4, 6, 6 5, 5, 6	3 3	6	$\frac{1}{36}$
17	5, 6, 6	3	3	$\frac{1}{72}$
18	6, 6, 6	1	1	$\frac{1}{216}$
total			216	1

Paving Slabs Investigation



1 Measure and record the length of the stick you are using.
2 Drop the stick onto the squared "paving slabs" area.
The stick may fall more "randomly" if you gently throw it upwards and let it fall back to the paper.
3 Record the number of times the stick falls across a line for 100 throws.
4 Work out the *relative frequency* of "hits".
5 What information does this answer give you?

Working Out Probabilities

There are different ways of arriving at probabilities.

	Method	*Example*	*Exact or Estimate?*
1	**Experiment** Do it lots of times, calculate the ***relative frequency*** and use that as an estimate for the probability.	Finding the probability of a drawing pin landing with its point upwards.	Estimate.
2	**Research** Use data already collected by someone else to calculate the ***relative frequency*** and use that as an estimate for the probability.	Finding the probability of someone who is left-handed passing their driving test first time.	Estimate.
3	**Personal Judgment** Give your best guess for the particular circumstances.	Finding the probability that a particular person will eat chips for lunch tomorrow.	Estimate.
4	**Theory** Decide that some outcomes ought to be equally likely and divide the number of equally likely ways it could happen by the total number of ways.	Finding the probability of throwing a 5 with a normal dice.	Exact if our assumptions are valid.

Think of some more examples for each of the four methods.

What are the advantages and disadvantages of each method?