3.6 Numeracy Ideas

- These ideas vary considerably in content and length of time necessary. Some might be useful as lesson starters/finishers. Others could develop into a whole lesson's work. Some would operate well as homework tasks. They are presented below in no particular order (deliberately, so as to promote some variety if you are working your way through them).
- Some tasks may become favourites with pupils and can be used again and again; others will just become boring if over-used: it's a matter for your judgement.

3.6.1 **NEED** acetate of one or more of the "Number Grids" pages.
Put one up on an OHP (this is economical, because you can use the same ones again and again). Pupils draw a 4 × 4 grid (make the squares reasonably large). When everyone's ready, issue a rule, e.g., "double it", or "divide by 100" and pupils write out the answers against the clock.

There are endless possibilities; e.g., rounding, finding the mean of a column, factors (with integers), etc.

Various grids are given:
1.Positive integers
2.Positive and negative integers
3.Positive decimals
4.Positive and negative decimals
5.Fractions
6.Percentages

Or you can use the grids orally – pointing at a number, say, "what's 100 take away this one", "what's the nearest prime number to this one", etc.

3.6.2 Can you make 100 out of four nines?

Answer: $99+\frac{9}{9}$ *or [99.99], where brackets indicate rounding to the nearest integer.*

3.6.3 By adding 1 straight line to this, make it true.

10 10 10 = 9.50

Answer: 10 TO 10 = 9.50
("ten minutes to ten")

3.6.4 Using the digits 1 to 9 once each, *in order*, how can you make 100?

Answers: lots of ways!
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 × 9 = 100
1 + 2 + 3 – 4 + 5 + 6 + 78 + 9 = 100
123 – 4 – 5 –6 – 7 + 8 – 9 = 100
1 + 2 × 3 + 4 × 5 – 6 + 7 + 8 × 9 = 100
12 + 34 – 5 + 6 + 7 + 8 – 9 = 100
and many more!

3.6.5 A nine-digit number has all its digits different. When it's multiplied by 8, the answer again has nine digits, all of them different. What are the numbers?

Answer: 123456789 × 8 = 987654312
(Notice the order of the final two digits.)

3.6.6 What **word** goes in the gap?
"This sentence has ______ letters."
e.g., the word "thirteen" won't do, because the sentence would have 30 letters.

Answer: "thirty-one" or "thirty-three"

Don't count a space or a full stop as a "letter".

3.6.7 **NEED** acetate of musical composers and the accompanying sheet.
Mental number work together with getting information from a list.

Pupils could research a version involving famous mathematicians or other famous people of interest to them. (With modern-day celebrities who are still alive, pupils could make up something based on their dates of birth.) The internet is ideal for collecting this sort of information.

3.6.8 **NEED** acetate of price list from the school canteen
"How much would it cost for …?"
"How much change would I get from a £5 note if I bought …?", etc.

3.6.9 Missing Digits.

Letters stand for digits (e.g., A = 7, B = 6, C = 0); within a question the same letter always represents the same digit.

1. Addition/Subtraction; e.g., 43A + B4 = 5C1 (usually best solved by writing in columns)
2. Multiplication (usually best solved by writing as long multiplication)
3. Division (usually best solved by converting into the equivalent multiplication problem)

Pupils can make these up. There will often be more than one possible answer.

You can play these like "hangman".

3.6.10 Target Numbers.
Pick four, five or six numbers, combine them in some way to make a "target number". Write the target number on the board in a circle.
e.g., numbers 11, 7, 3 and 9 and the target number is 35.

Answer: $11\times3-7+9$ *= 35*

Pupils may need to know BIDMAS, or you could use this task to introduce it.

The rule can be either that you have to use all the numbers or that you don't.
Normal maths symbols like +, – , etc. are always allowed.

You can use a today's date or a pupil's date of birth as the digits; e.g., 12th January 1992 gives 1, 2, 0, 1, 9, 2 (12/01/92) as the six numbers.
Gradually work around the whole class.

You can choose any reasonable target number, since if it is impossible to make exactly then the winner will be the person who gets the nearest.

3.6.11 Mental Squares.
Draw this on the board.

1	2	3
4	5	6
7	8	9

You've got 15 seconds to memorise it, then I'm going to rub it off. Then I'm going to ask you some questions about it.

(Press hard with the board rubber so that the numbers don't still show!)

1. What number is in the middle?
2. Add up the right hand row.
3. Multiply the three numbers on the top row.

Some pupils may prefer to close their eyes when thinking about the questions.

You can change the square by saying "Add 9 to the middle number. Now what is the total of the middle column?" or "Swap the numbers at the top right and bottom left. Now what is the total of the bottom row?", etc.

You could write R and L on the right and left sides of the board to make the task accessible to pupils who muddle up the directions.

3.6.12

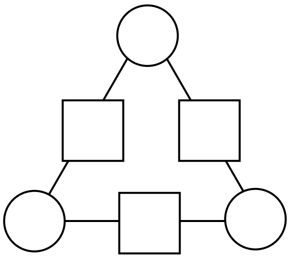
- Find a number which is increased by 12 when it's turned upside down.
- Find a number which is increased by 21 when it's turned upside down.
- Find a number which is twice the product of its digits.
- Find a number that is the same when turned upside down. How many are there?

Answers:
86
68
36
69, 6699, 88, 111, 101, etc.

3.6.13 Number Triangles (see sheets).

Easy to make up.

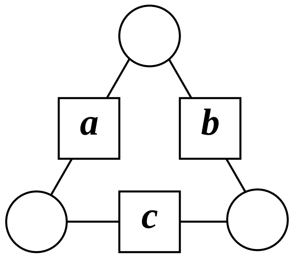
Methods of solution for the different rules:



Generally you give the numbers in the square boxes and the task is to find what the numbers in the circles have to be.

There are different possible rules:

1. Addition:
   "Square number" = sum of the two adjacent "circle numbers";
2. Multiplication:
   "Square number" = product of the two adjacent "circle numbers".



Once you have found one "circle number", the rest are easy to calculate.
So to find the "circle number" at the top,

1. *top number =* $\frac{1}{2}(a+b-c)$ *;*
2. *top number =* $\sqrt{\frac{ab}{c}}$ *, assuming that an odd number (one or three) of* a *,* b *and* c *are not negative, and that* $c \neq 0$ *.*

3.6.14 Bingo.

Method 1 (quicker):
Draw a 5 × 5 square and fill it with the numbers from 1 to 25 (each one once each).
(It may be easiest to write them down in numerical order so that you know you haven't missed one.)
You need to get all 5 numbers in a row or a column or a diagonal to win. If an answer lies outside the range 1 to 25, ignore it (but don't say anything otherwise you'll help other people to win!).

Method 2 (longer):
Draw a 2 × 5 rectangle.
Choose 10 different integers between 1 and 50 and fill them into the 10 squares however you like.
You need to get a "full house" to win.
If an answer lies outside the range 1 to 50, ignore it.
(It's probably best not to allow repeated numbers, because it can cut the game very short; someone could win on the first question.)

You could give out prizes if you're feeling generous!

For "times tables bingo" you can have a bag containing small pieces of card with the numbers 1 to 12 written on (two of each number). You pull out two cards, and that's your multiplication. Try to vary the order in which you say the two numbers (e.g., "6 × 9" as well as "9 × 6") and use words like "product", "times", "multiply", "lots of", etc.

Just normal mental number work; e.g.,

1. *a number of identical items of a certain value bought for a given total cost;*
2. *converting between units of length / area / volume, etc. (metric / imperial, etc.);*
3. *polygon or angle facts (e.g., "the number of degrees inside a quadrilateral divided by the number of sides in a hexagon all divided by 10");*
4. *powers and roots (e.g., "the cube root of 125 take-away the positive square root of 16");*
5. *multiples and factors (e.g., "the LCM of 12 and 8 divided by the HCF of 12 and 8");*
6. *decimal and fraction multiplication; percentages;*
7. *negative numbers;*
8. *square / triangle numbers, etc. (e.g., "the third cube number divided by the second triangle number");*
9. *algebra (e.g., "if* $a=4$ *and* $b=10$*, what is* $ab-2b$ *");*
10. *inequalities (e.g., "if* $2x>15$ *and* $x<9$*, and* x *is an integer, what is* x *?"*
11. *mean, mode, median, range.*

If necessary, pupils can keep an eye on the people sitting near to them to make sure they don't alter their numbers once the game has started! It may be worth keeping a record of the answers to your questions in case of any dispute over which numbers have "gone".

A cloth money bag, available from banks, is ideal for this.

3.6.15 Numbers and Words (see sheet).
Answers:
Task 1:

Some suggested tasks to try to make this potentially dull topic more interesting.

You always eventually hit "four", which has 4 letters.

A creative pupil could perhaps adapt task 1 to make a magic trick.

Task 2:
The largest I can do is FIVE THOUSAND.
The smallest I can do is MINUS FORTY.

Pupils may be able to do better than these.

Task 3:
The largest is 17 and the smallest is 1. In order they go 1, 10, 5, 9, 8, 6, 3, 2, 4, 11, 7, 15, 18, 19 12, 16, 13, 14, 20, 17 (smallest to largest).

Word Values:

number	value	number	value
ONE	34	ELEVEN	63
TWO	58	TWELVE	87
THREE	56	THIRTEEN	99
FOUR	60	FOURTEEN	104
FIVE	42	FIFTEEN	65
SIX	52	SIXTEEN	96
SEVEN	65	SEVENTEEN	109
EIGHT	49	EIGHTEEN	73
NINE	42	NINETEEN	86
TEN	39	TWENTY	107

This task is in danger of reinforcing the misconception that A always equals 1, B always equals 2, and so on. Yet this is an enjoyable activity which it would be a pity to omit.

Values close to 100: *Equation (102), polygon (104), formulas (105) and mathematical (106) are all close.*

(See sheet of strips of alphabet and code.)

You can insist that words must be maths-related or not, as appropriate.

3.6.16 Integer Investigations (see sheet).

Some interesting tasks involving integer arithmetic.

Answers:

Process 1:
You always get 1089.
4-digit numbers give 10890
5-digit numbers give 109890, and so on
(6 gives 1098900, 7 gives 10998900, etc.).

1089 Proof
If the original number is "abc", we can write that as $100a+10b+c$*, and we'll assume that* $a>c$ *so that this is bigger than its reverse.*
The reverse number can be written $100c+10b+a$*, and when we subtract the smaller from the bigger we get* $100(a-c)+(c-a)=99(a-c)$*, so at the first stage we will always get a multiple of 99.*
Since $a>c$*, the "units digit"* $(c-a)$ *is negative, so we have to break one of the hundreds into 90 + 10.*
This gives $100(a-c-1)+10\times9+(10+c-a)$*.*
The units digit $(10+c-a)$ *is now positive.*
When we reverse this number we get $100(10+c-a)+10\times9+(a-c-1)$*, and doing the addition gives us* $100(10-1)+2\times10\times9+(10-1)$ $=1089$*.*
This only works because we assumed that a*,* b *and* c *were all different from each other.*

Process 2:
Within 6 steps you should reach 495.
This is Kaprekar's process *(1905-1988).*
With 4-digit numbers, you should reach 6174 within 7 steps.
It is very hard to explain completely why this happens.

Process 3:
e.g., *44 → 22 → 11 → 34 → 17 → 52 → 26 → 13 → 40 → 20 → 10 → 5 → 16 → 8 → 4 → 2 → 1.*
You always end up with a loop that goes 4 → 2 → 1 → 4 → 2 → 1, etc. You get there quickly once you hit a power of 2 (e.g., from 16 above).
This is called Collatz's process *(1910-1990).*
No-one knows if you will always get to 1 whatever number you begin with.

Process 4:
You always end up at 4 eventually, because that's the only number which makes itself when you add 8 and divide by 3.
If $x_{n+1}=\dfrac{x_n+8}{3}$*, then* $3x=x+8$*, so* $x=4$*.*

In general, if the rule is "add a*, divide by* b*", then the final number will be* $\dfrac{a}{b-1}$*. This won't work if* $b\le1$*, because the numbers in the sequence will just get bigger and bigger.*

3.6.17 Un-magic Squares (see sheets).

You can either use the sheets or just write a couple onto the board.

3.6.18 Boxes (see sheet).

A Fibonacci-type (1170-1250) investigation.

(This idea also appears in section 3.7.6.)

3.6.19 Picture Frames (see sheets).

These can be quite challenging. Pupils' own problems along these lines can be extremely difficult.*

3.6.20 Biggest Products (see sheet).

This task can be used as an excuse to practise non-calculator multiplication (e.g., by the gelosia method – see section 1.5). It is a way of giving a larger purpose to some routine practice and also incorporates some logical thinking, possibly even some algebra.

3.6.21 Scoring 100 (see sheet).

Numeracy games suitable for playing in pairs. They are an opportunity to develop number skill but also to think about strategy.
Pupils enjoy being "in the know" and winning with ease once they know how!

3.6.22 Broken Calculator (see sheet).

This is a non-*calculator task!*

It is an interesting task to consider which buttons on the calculator are "really essential" and which are just "convenient". There isn't a sharp distinction, since even trigonometrical functions can be evaluated using power series, but it is reasonable to say, for example, that we could manage with just the $\sin$ *button and use identities like* $\cos x = \sin(90 - x)$ *to find* $\cos$ *of anything and then use* $\tan x = \dfrac{\sin x}{\cos x}$ *to find* $\tan$ *of anything.*
Which other buttons could we manage without?

Another way of putting it is "if you could have only five buttons beyond the numbers 0 to 9, what five would you choose?" *(It does depend to some extent on the kind of calculations the pupils would anticipate needing to do.)*
- *Do two presses of the "minus" key work as addition?*
- *Do you need the decimal point AND the divide key?*
- *The power key can cover "squared" and all the roots.*

This task makes you appreciate a fully-functional calculator!

3.6.23 Day of the Week (see sheet).

An interesting task involving careful following of instructions and simple number calculations.

Pupils may not know that British elections happen on Thursdays, but they should still be able to identify that one by a process of elimination.

Other dates that pupils/parents may remember the day of might include the following:
- assassination of President Kennedy:
  Friday 22 November 1963;
- Neil Armstrong stepping onto the moon:
  Monday 21 July 1969;
- wedding of Prince Charles and Lady Diana:
  Wednesday 29 July 1981;
- death of Princess Diana:
  Sunday 31 August 1997;
- terrorist attacks in US:
  Tuesday 11 September 2001.

Answers:
Obviously there may be links with material pupils are studying in History.

"Black Friday"	18 November 1910
"Bloody Sunday"	22 January 1905
"Black Wednesday"	16 September 1992
UK General Election	1 May 1997

1910 was the Suffragettes' demonstration outside the Houses of Parliament;
1905 was the Russian demonstration (not the Irish "Bloody Sunday");
1992 was the fall of the pound from the exchange rate mechanism;
1997 was the big Labour win.

31	22	17	44	6
10	57	2	26	68
79	72	45	12	0
94	3	63	1	19
81	20	98	5	38

4	**2**	**12**	**–1**
1	**–7**	**32**	**–16**
–3	**28**	**0**	**–5**
26	**–2**	**15**	**–30**

0.01	3.5	2.08	3.17
5.6	0	4.3	0.25
5.08	1.3	6.2	0.1
2.8	10.3	1.99	4.95

0.3	**1.83**	**6.66**	**–0.12**
0.9	**–3.55**	**0**	**4.4**
1.7	**–0.05**	**5.9**	**–0.1**
–2.06	**–10.3**	**–5.37**	**2.61**

$\frac{2}{3}$	$\frac{9}{10}$	$\frac{1}{6}$	$\frac{2}{5}$
$\frac{10}{21}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
$\frac{4}{5}$	$\frac{5}{6}$	$\frac{2}{11}$	$\frac{1}{3}$
$\frac{1}{4}$	$\frac{4}{9}$	$\frac{3}{5}$	$\frac{99}{100}$

20%	**15%**	**11%**	**75%**
24%	**7%**	**1%**	**30%**
2%	**65%**	**50%**	**80%**
60%	**40%**	**5%**	**25%**

Bach	1685-1750
Handel	1685-1759
Haydn	1732-1809
Mozart	1756-1791
Beethoven	1770-1827
Schubert	1797-1828
Mendelssohn	1809-1847
Chopin	1810-1849
Schumann	1810-1856

Musical Composers TEACHER'S NOTES

Bach	**1685-1750**
Handel	**1685-1759**
Haydn	**1732-1809**
Mozart	**1756-1791**
Beethoven	**1770-1827**
Schubert	**1797-1828**
Mendelssohn	**1809-1847**
Chopin	**1810-1849**
Schumann	**1810-1856**

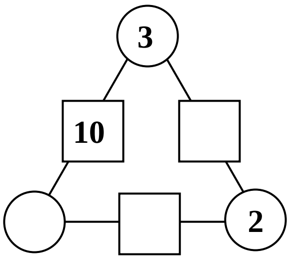
Cover up left side: "What do you think these numbers are?"
(Are they likely to be take-away sums? What makes them look like dates?, etc.)

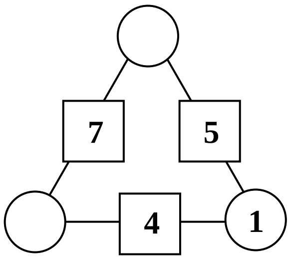
Say that it's OK for pupils to abbreviate the composers' names when writing their answers, so long as they find a way of distinguishing between names that start with the same letter.

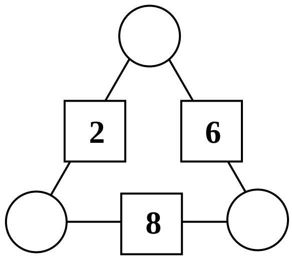
1	How long did Bach live?	***65 years***
2	How long did Haydn live for after Mozart had died?	***18 years***
3	Which people on the list could Mozart have met if they'd been in the same place?	***Handel, Haydn, Beethoven***
4	How much longer did Handel live than Beethoven?	***17 years longer***
5	Who lived the longest?	***Haydn***
6	Who died the youngest?	***Schubert***
7	When will it be the 300th anniversary of Haydn's birth?	***In 2032***
8	We were all born in the 20th century and we'll probably all die in the 21st century. Which people on the list were like us – born in one century and died in the next?	***Bach, Handel, Haydn, Beethoven, Schubert***
9	Which composer lived 1 year longer than another one on the list?	***Chopin (1 year longer than Mendelssohn)***
10	I'm going to tell you about a composer who *isn't* on the list. His name is Wagner and he lived twice as long as Mozart. He was born in 1813. When did he die?	***1883***

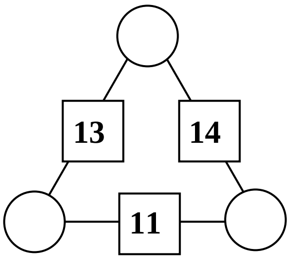
Number Triangles (Addition)

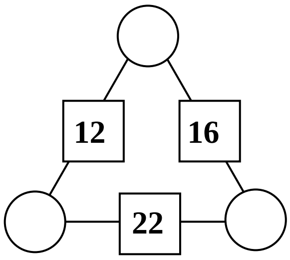
The number in each square is the *sum* of the numbers in the two circles connected to it.

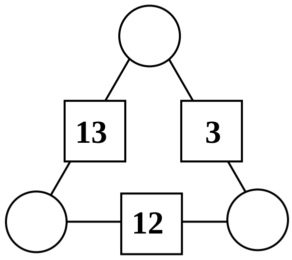


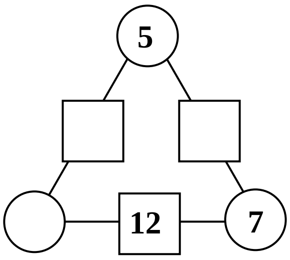


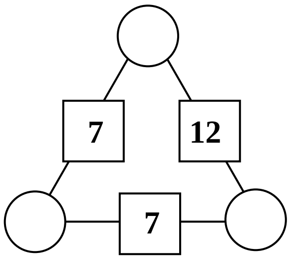


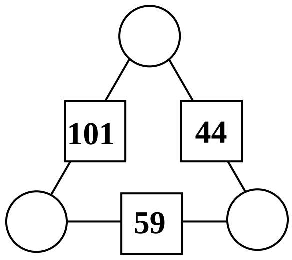


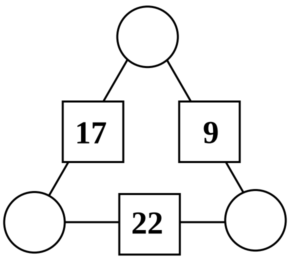


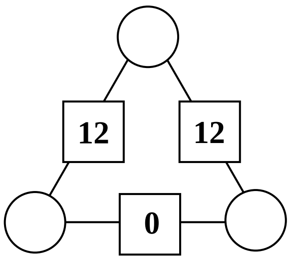


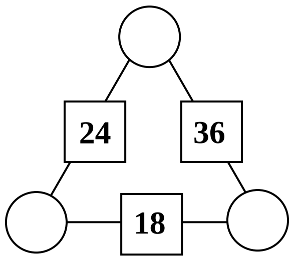


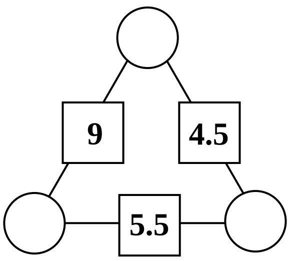


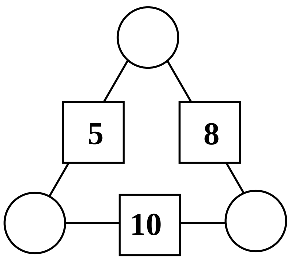


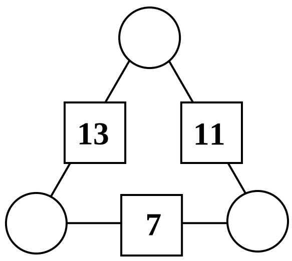






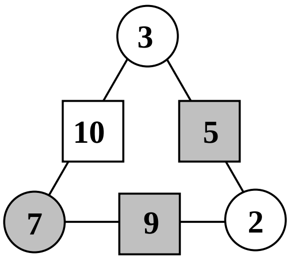


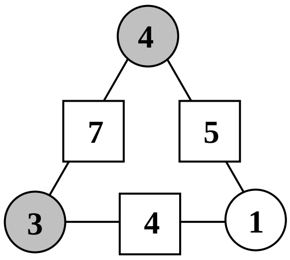


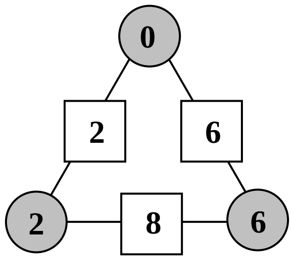


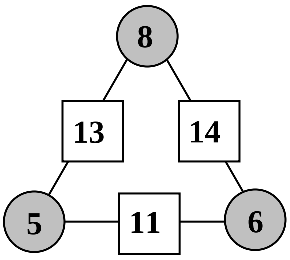
Number Triangles (Addition) ANSWERS

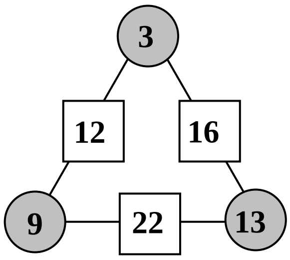
The number in each square is the *sum* of the numbers in the two circles connected to it.

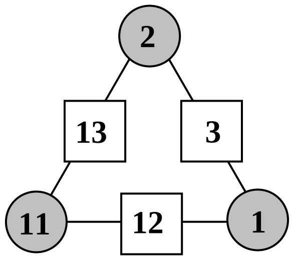


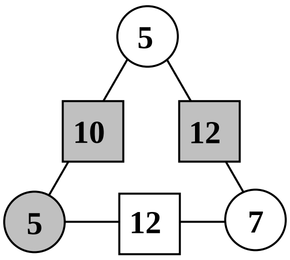


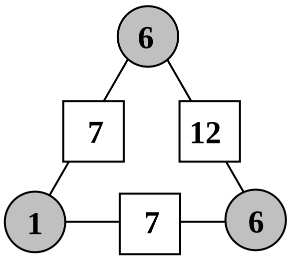


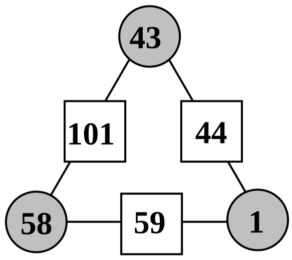


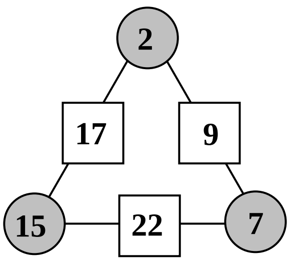


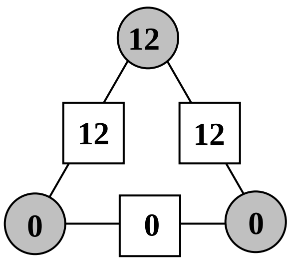


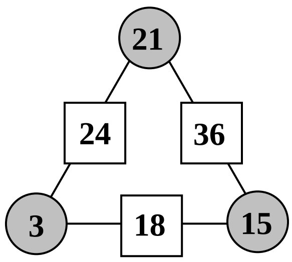


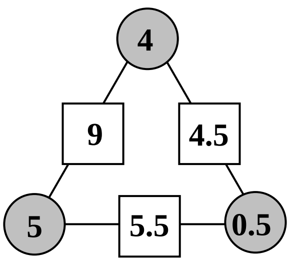


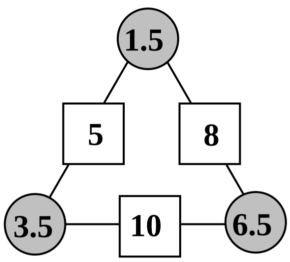


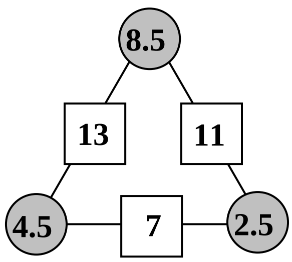






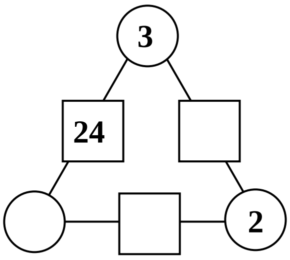


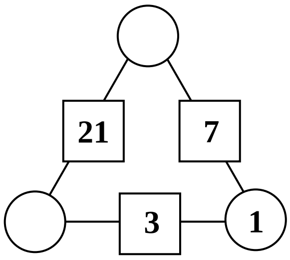


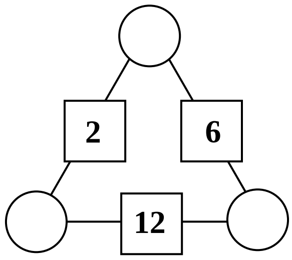


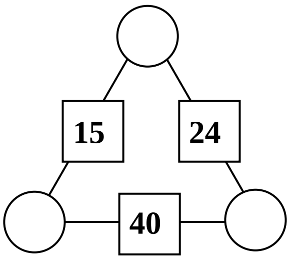
Number Triangles (Multiplication)

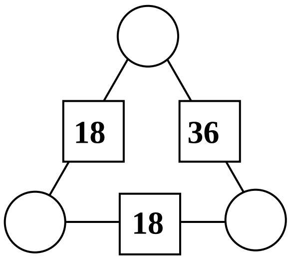
The number in each square is the *product (multiplication)* of the numbers in the two circles connected to it.

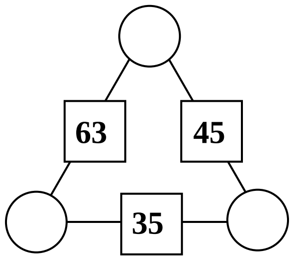


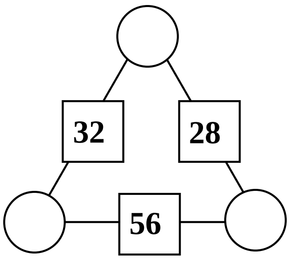


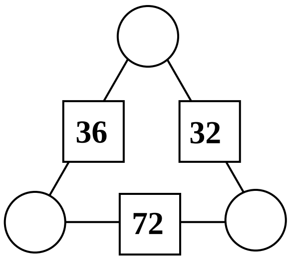


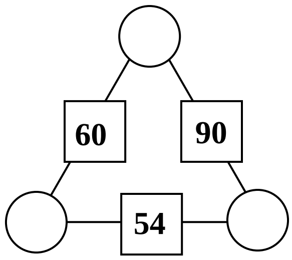


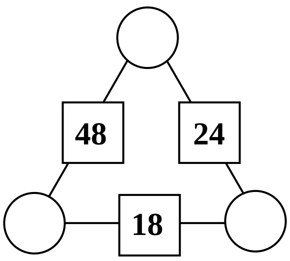


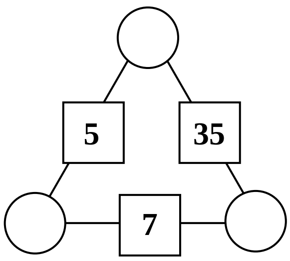


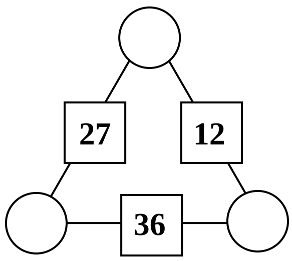


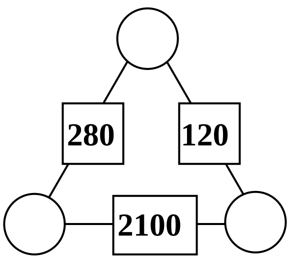


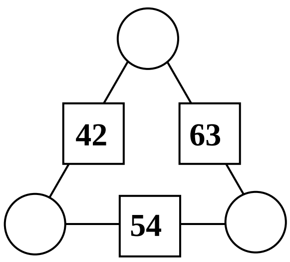


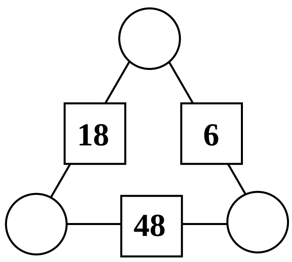






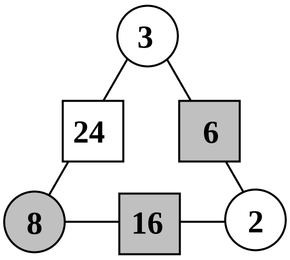


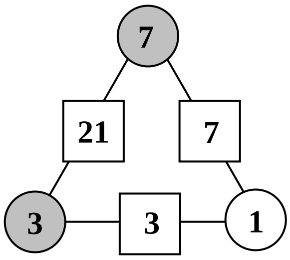


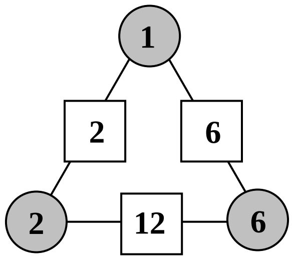


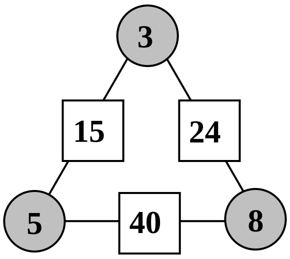
Number Triangles (Multiplication) **ANSWERS**

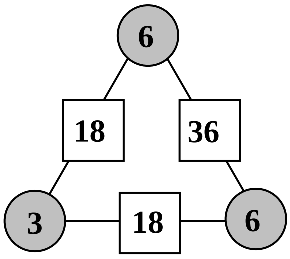
The number in each square is the *product (multiplication)* of the numbers in the two circles connected to it.

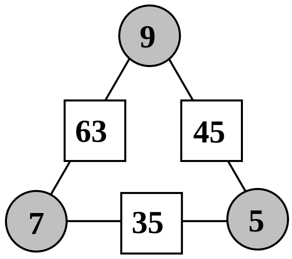


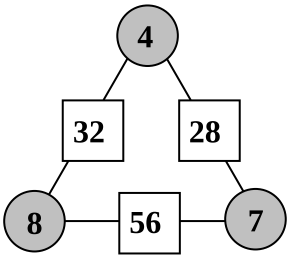


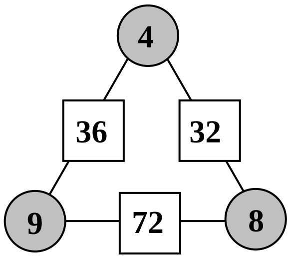


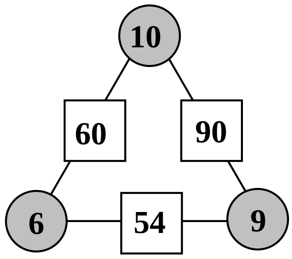


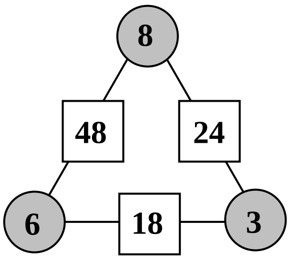


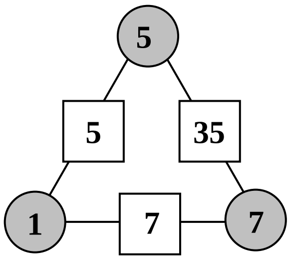


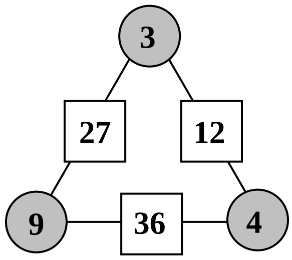


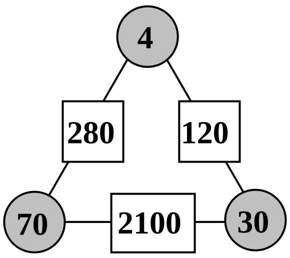


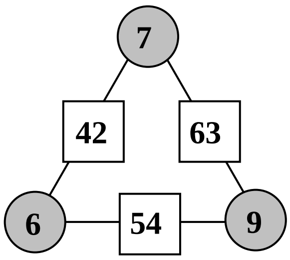


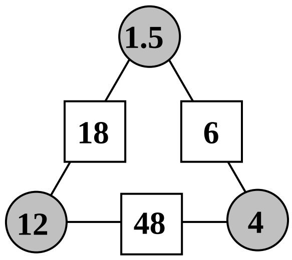












Numbers and Words

Remember that there is no "U" in FORTY.

Task 1

Choose an integer between 1 and 100 and write it in words; *e.g., thirty-eight*

Count up the total number of letters; *e.g., 11*

Write this number in words; *e.g., eleven*

Keep going; *e.g., six, three, …*

What happens eventually?

Try it with some different starting numbers.

Can you explain why this happens?

Task 2

The three letters in the word *ten* are all different.
What is the largest number you can write in words where all the letters are different? You can use any letters you like, but you can use each letter only once.

What is the smallest number you can do?

Task 3

Let A = 1, B = 2, C = 3, etc. for this task.

For a word, like MATHS, the value is the *sum* of the values of the letters that make it up.
e.g., for MATHS, 13 + 1 + 20 + 8 + 19 = 61.

Which 5-letter word has the biggest total?
It has to be a proper word.

Which has the smallest total?

Which number (in words) between *one* and *twenty* do you expect to have the biggest value? Which do you think would have the smallest?
Try it and see.

Write the numbers *one* to *twenty* in order of their word values.

Find a word with a value as close to 100 as you can.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Integer Investigations

Try these processes and see what happens in each case.
Follow the instructions carefully.

Process 1

		example
1	Choose any 3-digit number where all the digits are different.	*375*
2	Write down the digits in the opposite order.	*573*
3	Subtract the smaller from the larger.	*573 – 375 = 198*
4	Write down the digits of this new number in the opposite order.	*891*
5	Add these last two numbers.	*198 + 891 = 1089*

Try some more numbers.
Describe what happens.
Can you explain why?
What if you start with a 4-digit or 5-digit number?

Process 2

		example
1	Choose a 3-digit number where all the digits are different.	*452*
2	Arrange the digits so that they go from biggest to smallest (left to right).	*542*
3	Arrange the digits so that they go from smallest to biggest.	*245*
4	Subtract the second one from the first one.	*542 – 245 = 297*
5	Repeat using the new number.	
6	Keep going until you have a good reason to stop.	

Describe what happens.
Can you explain why?
Try it with 4-digit numbers.

Integer Investigations (continued)

Try these processes and see what happens in each case.
Follow the instructions carefully.

Process 3

		example
1	Choose any 2-digit number.	*38 → 19 → 58 → 29 → 88 → etc.*
2	If the number is even, divide by 2.	
3	If the number is odd, multiply by 3 and add 1.	
4	Go back to step 2.	
5	Keep going until you have a good reason to stop.	

Describe what happens.
Can you explain why?

Process 4

		example
1	Choose any integer between 1 and 100.	*47*
2	Add 8.	*55*
3	Divide the answer by 3. (If you get a decimal, that's OK.)	*18.333…*
4	Go back to step 2.	*26.333…*
5	Keep going until you have a good reason to stop.	*8.778…, etc.*

Describe what happens.
Can you explain why?

Make up a similar rule.
What happens this time?

 # Un-Magic Squares!

In a *magic square*, all the rows, all the columns and both diagonals add up to the same amount (the magic total).

The squares below are ***un**-magic squares*; there's *one wrong number* in each one.

Find the wrong number and correct it.

11	26	5
8	14	20
23	2	18

magic total =

12	22	20
25	18	10
16	14	24

magic total =

96	19	74
41	59	85
52	107	30

magic total =

38	93	16
27	49	71
82	6	60

magic total =

43	52	15
8	36	64
57	22	29

magic total =

28	24	45
48	32	16
20	40	36

magic total =

29	44	23
26	32	38
41	21	35

magic total =

25	20	21
18	22	26
32	24	19

magic total =

21	35	25
31	27	23
29	19	34

magic total =

147	143	163
167	151	153
139	159	155

magic total =

153	104	181
170	148	126
115	192	137

magic total =

289	224	237
197	250	302
263	276	211

magic total =

Un-Magic Squares! ANSWERS

In a *magic square*, all the rows, all the columns and both diagonals add up to the same amount (the magic total).
The squares below are ***un**-magic squares*; there's *one wrong number* in each one.
Find the wrong number and correct it.

One way of working is to write the totals of each row/column/diagonal round the edge. Sometimes you may notice the incorrect number because it is the only odd or only even number.

The shaded boxes are the corrected numbers, and the magic totals are in bold underneath.

11	26	5
8	14	20
23	2	**17**

magic total = **42**

12	22	20
26	18	10
16	14	24

magic total = **54**

96	19	74
41	**63**	85
52	107	30

magic total = **189**

38	93	16
27	49	71
82	**5**	60

magic total = **147**

43	**50**	15
8	36	64
57	22	29

magic total = **108**

28	24	**44**
48	32	16
20	40	36

magic total = **96**

29	44	23
26	32	38
41	**20**	35

magic total = **96**

25	20	21
18	22	26
23	24	19

magic total = **66**

21	35	25
31	27	23
29	19	**33**

magic total = **81**

147	143	163
167	151	**135**
139	159	155

magic total = **453**

159	104	181
170	148	126
115	192	137

magic total = **444**

289	224	237
198	250	302
263	276	211

magic total = **750**

Boxes TEACHER'S NOTES AND ANSWERS

Draw a line of 5 boxes on the board with the numbers 3 and 4 in the first two boxes.

3	4			

The rule is that from the third number onwards, the number in each box is the sum of the *two* previous numbers. (Note, *not* the sum of *all* the previous numbers, just the previous *two*.)
(It's clearer to use the word "previous" because the "last" number may be taken to mean the number in the far right box.)
So we get

3	4	7	11	18

Doing it this way is pretty easy, but if I just gave you

3				18

and you had to find the missing numbers it would be much harder.

Try these. You can assume that all the numbers are positive integers, and that the number in the second box is larger than the number in the first box.

a

8	**12**	**20**	**32**	52

b

6	**10**	**16**	**26**	42

c

12	**17**	**29**	**46**	75

d

31	**43**	**74**	**117**	191

e

11	**15**	**26**	**41**	**67**	108

f

5	**12**	**17**	**29**	**46**	75

g

13	**20**	**33**	**53**	**86**	139

h

7	**15**	**22**	**37**	**59**	96

If instead I gave you

	15				96

are there any other possibilities (apart from the previous solution)? No.

What if I just gave you

					96

How many possible answers are there where all the numbers are positive integers?

The only other possibility where the 2nd number is bigger than the 1st is
2, 18, 20, 38, 58, 96.
If the 2nd number can be smaller than the 1st, then you can also have
2, 12, 24, 36, 60, 96;
22, 6, 28, 34, 62, 96;
27, 3, 30, 33, 63, 96;
32, 0, 32, 32, 64, 96.

Using algebra, if x and y are the integers in the first two boxes respectively ($y > x$), then the number in the fifth box will be $2x + 3y$, and the number in the sixth box will be $3x + 5y$.
(The co-efficients are the numbers from the Fibonacci sequence.) So you can repeatedly subtract 3 from 96, checking each time whether you have a multiple of 5;
e.g., 96 – 7 × 3 = 75 = 15 × 5, so x can be 7 and y can be 15 (the original set of boxes).

Picture Frames

A rectangular picture frame is broken into straight pieces.
The lengths of the pieces are measured.
How should the pieces be fitted back together to make the original picture frame?

The first 5 were *square* picture frames.

lengths of pieces of picture frame (inches)

1 1, 1, 2, 2, 2, 3, 4, 5

2 1, 4, 6, 6, 8, 11, 12

3 1, 3, 4, 4, 5, 6, 6, 7, 7, 7, 10

4 10, 20, 30, 40, 50, 60, 70, 80

5 2, 2, 3, 3, 4, 5, 6, 10, 12, 14, 15

Rectangular picture frames are harder.
Try these.

lengths of pieces of picture frame (inches)

6 1, 1, 2, 3, 4, 4, 7

7 2, 2, 3, 3, 4, 4, 5, 7

8 1, 1, 2, 4, 4, 5, 7

9 2, 4, 5, 5, 6, 7, 7

10 1, 2, 3, 3, 6, 8, 9, 12

11 3, 3, 4, 5, 7, 10, 10, 10

12 1, 2, 2, 2, 3, 3, 6, 7

13 3, 3, 4, 4, 4, 4, 5, 11, 14

14 5, 6, 7, 8, 10, 10, 10, 12, 14

15 4, 4, 5, 6, 6, 7, 7, 8, 15

Can you invent a systematic way of solving problems like these?

Picture Frames ANSWERS

A rectangular picture frame is broken into straight pieces.
The lengths of the pieces are measured.
How should the pieces be fitted back together to make the original picture frame?

The first 5 were *square* picture frames.
Here you can add up the lengths of all the pieces and divide by 4 to find out how long the sides have to be.

	lengths of pieces of picture frame (inches)	**length of sides (inches)**	
1	1, 1, 2, 2, 2, 3, 4, 5	1 + 2 + 2 1 + 4	2 + 3 5
2	1, 4, 6, 6, 8, 11, 12	1 + 11 4 + 8	6 + 6 12
3	1, 3, 4, 4, 5, 6, 6, 7, 7, 7, 10	1 + 7 + 7 3 + 6 + 6	4 + 4 + 7 5 + 10
4	10, 20, 30, 40, 50, 60, 70, 80	10 + 80 20 + 70	30 + 60 40 + 50
5	2, 2, 3, 3, 4, 5, 6, 10, 12, 14, 15	2 + 3 + 14 2 + 5 + 12	3 + 6 + 10 4 + 15

Rectangular picture frames are harder.
Try these.

	lengths of pieces of picture frame (inches)	**longer sides (inches)**	**shorter sides (inches)**
6	1, 1, 2, 3, 4, 4, 7	1 + 7 4 + 4	1 + 2 3
7	2, 2, 3, 3, 4, 4, 5, 7	2 + 7 4 + 5	2 + 4 3 + 3
8	1, 1, 2, 4, 4, 5, 7	1 + 2 + 4 7	1 + 4 5
9	2, 4, 5, 5, 6, 7, 7	4 + 7 5 + 6	2 + 5 7
10	1, 2, 3, 3, 6, 8, 9, 12	1 + 12 2 + 3 + 8	3 + 6 9
11	3, 3, 4, 5, 7, 10, 10, 10	3 + 3 + 10 4 + 5 + 7	10 10
12	1, 2, 2, 2, 3, 3, 6, 7	2 + 7 3 + 6	1 + 3 2 + 2
13	3, 3, 4, 4, 4, 4, 5, 11, 14	3 + 4 + 11 4 + 14	3 + 5 4 + 4
14	5, 6, 7, 8, 10, 10, 10, 12, 14	6 + 10 + 10 12 + 14	5 + 10 7 + 8
15	4, 4, 5, 6, 6, 7, 7, 8, 15	4 + 15 5 + 7 + 7	4 + 8 6 + 6

There may be other possible answers.

Biggest Products **TEACHER'S NOTES AND ANSWERS**

- Choose four positive integers between 1 and 9.
  Say 3, 4, 7 and 9.

- If you make two 2-digit numbers out of these four digits, and multiply them together, what's the biggest product you can make?
  For example, 34 × 79 = 2686, but can you do better?

  Answer:
  The biggest product comes from 93 × 74 = 6882.
  If you have four digits $a < b < c < d$, then you always need to do $"da" \times "cb"$.
  The logic here is that you want to put the two biggest digits (d and c) in the tens columns, and then b (the next biggest digit) must go with c (not d) so that it gets multiplied by the d in the tens column of the other number.

- What if you're allowed to make a 3-digit number. You're still allowed only one multiplication sign. Will that get you a bigger product?

  Answer: No.

- What if you have five digits; say, 2, 3, 4, 7 and 9.
  What's the biggest possible product now? You can use 1-digit, 2-digit, 3-digit or 4-digit numbers, but you may use each digit only once and you are allowed only one multiplication sign.

  Answer:
  The biggest product now comes from 742 × 93 = 69006.
  If you have five digits $a < b < c < d < e$, then you need to do $"dca" \times "eb"$.
  The logic here is that you want to put the two biggest digits (c and d) in the highest possible columns, and then c (the next biggest digit) must go with d (not e) so that it gets multiplied by the e in the other number.

Extra Task

Using the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 and one multiplication sign only, what is the largest product you can make?
Answer: 87531 × 9642 = 843973902

Scoring 100 TEACHER'S NOTES AND ANSWERS

Making 100.

In pairs.

- Player 1 chooses an integer between 1 and 10 (inclusive) and writes it at the top of a piece of paper.
- Player 2 then chooses an integer (again, between 1 and 10 inclusive), adds it to the number on the paper and writes the answer below the number player 1 wrote.
- The players continue, taking it in turns.
- The winner is the first person to reach 100 exactly.

Play it a few times.
Can you work out a strategy so that you stand the best chance of winning.
Can you be sure of always winning?

The best strategy is always to add on whatever you need to to make one of these numbers:
1, 12, 23, 34, 45, 56, 67, 78 or 89 (the units digit is always one more than tens digit).
Once you've got to any one of these numbers, you're certain to win, so long as you keep with this strategy.

(The teacher can play this strategy and "beat anyone". "How am I doing it?" So as not to make it too obvious, you can take a risk and play randomly until you get to about 50.)

If the other person doesn't know about this strategy and you follow it, you will usually win. If you start, then you should certainly win.

Avoiding 100.

In pairs.

The same game as above, but this time whoever makes 100 *loses*. (You have to force the *other* person to make 100.)

When the strategy to the above game has been determined, change the rules to make it into this game.

This time the numbers you have to aim for are 1 less; i.e., 11, 22, 33, 44, 55, 66, 77, 88 or 99 (the eleven times table).

This time if the ***other*** *person starts and you follow this strategy you should always win. This is a nice example of a situation where starting is a* disadvantage. *This happens because whatever number* you *start with, the other person (if they realise it) can always add on enough to make 11 (the first "magic number") and then keep with this strategy and win.*

What if the target were not 100 but a different number?
Which "magic numbers" would you have to aim for along the way?

Broken Calculator TEACHER'S NOTES AND ANSWERS

Imagine you have a simple non-scientific calculator, but it is broken.
The only buttons that will work are these ones:

3	4	–	×	=

The calculator's insides are OK, and it still gives correct answers.
You can read the display without any problem.

Which integers between 1 and 20 can you make appear on the display?

Are any impossible?

no.	one possible route	no.	one possible route
1	4 – 3	11	3 × 3 × 3 – 4 – 3 – 3 – 3 – 3
2	4 × 3 – 4 – 3 – 3	12	3 × 4
3	3	13	4 × 4 – 3
4	4	14	3 × 3 × 3 – 4 – 3 – 3 – 3
5	4 × 3 – 4 – 3	15	3 × 3 × 3 – 3 – 3 – 3 – 3
6	3 × 3 – 3	16	4 × 4
7	4 × 4 – 3 – 3 – 3	17	3 × 3 × 3 – 4 – 3 – 3
8	4 × 3 – 4	18	3 × 3 × 3 – 3 – 3 – 3
9	3 × 3	19	3 × 3 × 3 – 4 – 4
10	4 × 4 – 3 – 3	20	3 × 3 × 3 – 4 – 3

There are many other possibilities.

Try it with different calculator buttons.

Are some buttons more "valuable" than others?

Clearly the +/– button or the subtract button is necessary if we're to get to any negative numbers.

Day of the Week

Everyone knows their date of birth, but do you know which *day of the week* you were born on?
If you don't, you can work it out; if you do, you can check that this process works.

When you do the divisions, always find an *integer* answer; ignore any remainder, and round *down*; so even if it's 19.8571… you give the answer 19.

Let's take Albert Einstein's birthday, 14 March 1879, as an example.

1 Start with the century number (e.g., 19 for 19-something, 20 for 20-something), and divide it by 4. Whatever the *remainder* is, call it r; so we have 18 ÷ 4 = 4 remainder 2, so $r = 2$ for our date.

2 In the formula below,
d = the day number; so $d = 14$;
m = the month number (March = 3, April = 4, etc., except that January = 13 instead of 1 and February = 14 instead of 2); so $m = 3$;
y = the year within the century (so it's between 0 and 99); so $y = 79$.

3 Now work out

$$A = d + \frac{26(m+1)}{10} + y + \frac{y}{4} + 5r;$$

remembering with each division to round to the next integer *down*.

4 Now divide A by 7 and call the *remainder* x.

5 Convert x to the day of the week (Sunday = 1, Monday = 2, etc.)
This is the day that we're after.
So in our example we get

$$A = 14 + \frac{26 \times (14+1)}{10} + 79 + \frac{79}{4} + 5 \times 2$$
$$= 132 \text{ (rounding down)}$$
$$\frac{A}{7} = \frac{132}{7}$$
$$= 18\tfrac{6}{7}$$
$$\text{so } x = 6 \text{ (Friday)}$$

which is correct.

Try this out on your birthday or on today's date to check that it works.

Puzzle
Decide which of the dates goes with which description:

"Black Friday"	1 May 1997
"Bloody Sunday"	16 September 1992
"Black Wednesday"	18 November 1910
UK General Election Day	22 January 1905