GOLDEN OPPORTUNITIES FOR CREATIVITY!

Colin Foster

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Is mathematics a creative subject? I certainly believe so, yet it can be difficult to find mathematical tasks which give pupils the opportunity to work in genuinely creative and imaginative ways. To be creative, pupils need more than to be presented with several options to choose from – they need the space to develop and pursue original work and see their ideas take form.

I tried to provide some scope for this while working on volume with my Y9 class. I asked pupils to design some gold coins that would be worth various amounts of money. I suggested values of £10, £100 and £1000, but was happy for pupils to work on any value they wished. My only requirement was that I wanted the work to be fairly accurate: if you're going to be minting millions of these coins, you need to be using exactly the right amount of gold.

Before we began, I asked whether there was anything we needed to know. As expected, pupils requested the density of gold (19.32 g/cm^3) and its price. We looked on the internet [1], briefly discussing why the value varies with time, and discovered that the price that morning was £221.171 per Troy ounce (31.10 g).

Pupils set about calculating and sketching various ideas. Some worked on cuboids, some on traditional discs and others on more exotic designs. Some pupils began by calculating the necessary volumes, but then were unsure how to find, say, r and h (two unknowns) from a formula such as $V=\pi r^2 h$. The fact that volume was a function of more than one variable made it necessary to make a choice about h, say, and see what effect that had on r. Other pupils began by deciding on dimensions they thought would be approximately right and then calculating the volume, sometimes finding that they were orders of magnitude out. Sometimes this could be rectified by applying ratio, but the differences between linear, area and volume scale factors were a source of much discussion and investigation. Some used trial and improvement to reach acceptable values.

Many pupils discovered that using ratio was efficient when calculating the sizes of higher denomination coins that were mathematically similar to coins they had already designed; for instance, scaling up the linear dimensions by a factor of $\sqrt[3]{10}$ when going from a £100 coin to a £1000 one. When silver was suggested as an alternative material (density = 10.5 g/cm^3 and cost = £3.937 per Troy ounce that day), ratio again proved to be the quickest way to find the dimensions of the new coins. There was the possibility of constructing the coins from two or more pieces (like the UK £2 coins) or from alloys or in different currencies.

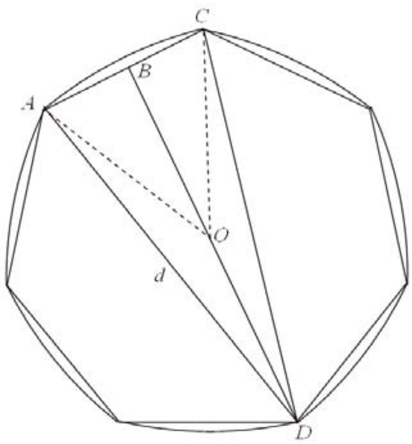
Issues raised along the way included the hazards (especially to pockets!) of coins with sharp corners, and the need for blind people and slot machines to be able to recognise the coins by their shape and feel. (Slot machines also appreciate coins of constant width, so that the orientation they have when they enter the machine doesn't matter.) An interesting question was the optimum balance between thickness and width: a coin that was wide and thin might bend or snap too easily, but one with more even dimensions might be too fiddly to pick up. No-one looked into properties like brittleness and hardness, although some commented on gold's beauty, non-toxicity and lack of tendency to corrode.

A problem that challenged me was to find the dimensions of a suitable coin in the shape of a 50 pence piece, a Reuleaux curve (also known, for polygons with odd numbers of sides, as a *curve of constant width*). It consists of seven identical arcs of a circle (like AC, see the figure), each with radius equal to the *diameter* d of the coin (AD or CD).

Finding the area of such a shape takes a bit of work:

Imagine a circle centred on O with radius OA.

The points A, C and D will all lie on the circumference. (Note that the arc AC is *not* an arc of *this* circle, but of a much larger circle with radius d and centre D.)



Since $A\hat{O}C = \frac{360°}{7}$, then by the circle theorem,

$A\hat{D}C = \frac{1}{2}A\hat{O}C = \frac{180°}{7}$

Area of segment ABC = area of sector ACD – area of triangle ACD

$$= \frac{180}{7} \div 360 \times \pi d^2 - \frac{1}{2} AC \times BD$$

$$= \frac{1}{14} \times \pi d^2 - \frac{1}{2} \times 2 \times d \sin A\hat{D}O \times d\cos A\hat{D}O$$

Since $A\hat{D}O = \frac{1}{2}A\hat{D}C = \frac{90°}{7}$, we have

Area of segment $ABC = \frac{\pi d^2}{14} - d^2 \sin\left(\frac{90}{7}\right) \cos\left(\frac{90}{7}\right)$

Therefore,

Area of coin = 7 × area of segment ABC + 7 × area of triangle OAC

$$= 7\left(\frac{\pi d^2}{14} - d^2 \sin\left(\frac{90}{7}\right) \cos\left(\frac{90}{7}\right)\right) + 7 \times \frac{1}{2} AC \times OB$$

$$= 7\left(\frac{\pi d^2}{14} - d^2 \sin\left(\frac{90}{7}\right) \cos\left(\frac{90}{7}\right)\right) + 7 \times \frac{1}{2} \times 2 \times d \sin A\hat{D}O \times \frac{AB}{\tan A\hat{O}B}$$

Since $A\hat{O}B = \frac{1}{2}A\hat{O}C = \frac{180°}{7}$, we have

Area of coin

$$= 7\left(\frac{\pi d^2}{14} - d^2 \sin\left(\frac{90}{7}\right) \cos\left(\frac{90}{7}\right)\right) + 7d \sin\left(\frac{90}{7}\right) \times \frac{d \sin\left(\frac{90}{7}\right)}{\tan\left(\frac{180}{7}\right)}$$

$$= \frac{\pi d^2}{2} - 7d^2 \sin\left(\frac{90}{7}\right)\cos\left(\frac{90}{7}\right) + \frac{7d^2 \sin\left(\frac{90}{7}\right)\cos\left(\frac{90}{7}\right)}{\tan\left(\frac{180}{7}\right)}$$

$= 0.7719d^2$ (correct to 4 significant figures).

Notice that this is just under $\frac{\pi d^2}{4} = 0.7854d^2$ (to 4 s.f.), which would be the area of a circle with centre O passing through A, C and D.

Clearly the method could be used for other odd-sided Reuleaux polygons.

And with just a little more work I had designed my gold coin.

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Reference

1 http://www.lbma.org.uk/2004dailygold.htm

MT editors request your assistance

The December issue of this journal will be the last MT (193) in its present form. Coincidentally it will also be the 50th anniversary of Mathematics Teaching. It would be wonderful to make this a grand celebration of the success of MT over those fifty years. If any of you were interested in sending us articles, short or long, puzzles, teasers etc. we would be grateful and more than pleased to publish them. Maybe there are pieces that have appeared in MT that have had a profound affect upon your teaching of mathematics and indeed your whole approach to mathematical thinking. Tell us about these as we would be very happy to reproduce them either on the website or in the journal itself.

It will be a time to look back and perhaps more importantly to look to the future and we hope the contents of MT193 will reflect this. Maybe some of you would be interested to play around with the numbers 1955 (the year of the first issue), 193 or maybe even 50. It would be fun to publish the results of your deliberations and maybe others will be motivated and respond to your findings.

All three of us look forward to receiving your ideas and articles which will ensure that the December issue is hugely successful.

Geoff Dunn, Robin Stewart & Helen Williams