MATHEMATICAL BEHAVIOUR

Colin Foster suggests that important mathematical ways of working are sometimes labelled as 'bad behaviour'.

I used to think of behaviour management as a separate entity from the teaching and learning of mathematics. I would do the maths teaching bit, switching over to the discipline bit as and when necessary, hoping to return to being a maths teacher again as quickly as possible. With more difficult classes, the two might run in parallel, yet the parallel lines would, of course, never meet. But I no longer see it quite that way. I now believe that issues of behaviour in the mathematics classroom are intimately connected to views of mathematics and the learning of mathematics. Maths teachers' perspectives on classroom behaviour will be influenced by many factors, but one often neglected and highly significant factor will be their view of mathematics as a subject.

To illustrate what I mean, I once observed a lesson in which a teacher, for the purposes of some kind of number trick, asked a pupil to give her 'a number between 1 and 10'. The pupil offered 'pi', and received the angry response: "Why do you always have to spoil things by being *silly*?" I wanted to answer, "Because that's what mathematicians *do*!" It seemed to me that 'pi' was a rather interesting and inventive answer, whatever that particular pupil's motives in offering it at that moment might have been. Actions such as 'pushing the boundaries', 'seeing what you can get away with', 'going as far as you can', 'trying to provoke a surprise', 'picking holes in a statement', 'looking for the exceptions', and so on, are generally seen as undesirable modes of behaviour in the school environment, yet they are all *highly* mathematical ways of operating. Could mathematics exist without these things? Surely we should be actively working to make space for behaviours like these to grow in the mathematics classroom, however inconvenient they might be at times.

Handling what might be termed 'deliberate awkwardness' is, I think, a particular issue for mathematics teachers. (I know very well that young people can be 'awkward' in virtually any context, but the absolute nature of mathematical statements almost invites it on occasion.) Most maths teachers will have examples of questions or statements they made that were deliberately misinterpreted by a smart pupil. I remember setting a list of expressions such as $10a - 6b$ underneath the instruction 'Factorise' and receiving:

$1(10a - 6b)$
$1(4c + 2d)$
...

which I now think shows rather a nice economical approach and an awareness of the unique identity property of unity. (See also a novel take on 'expand' on page 13.) Indeed, I had probably said many times to the class that 'Mathematicians are lazy!' and 'Read the question!' and 'Don't do unnecessary work!' and so on. So what was wrong with this answer? With hindsight, I am rather glad that he exploited my failure to say 'Factorise *fully*', but, in the hurly-burly of the classroom, a teacher's immediate reaction can be quite different!

Some time ago, I was doing a 'people maths' lesson outside and I wanted pupils to stand along an angle bisector ("Get so that you're the same distance from this wall as from this wall"). A particularly 'kinæsthetic' pupil ran along the desired line, away from the vertex, shouting, "I'm OK here; I'm still OK here; I'm still right now; Look, I'm still right!", disappearing across the school field. It annoyed me a bit at the time – but then I suppose I could argue that getting annoyed with unanticipated 'infinities' is also a highly mathematical tendency! This action was certainly a memorable image for everyone that an angle bisector 'goes on forever'. Would I have been happier if this pupil had adopted a more conventional position

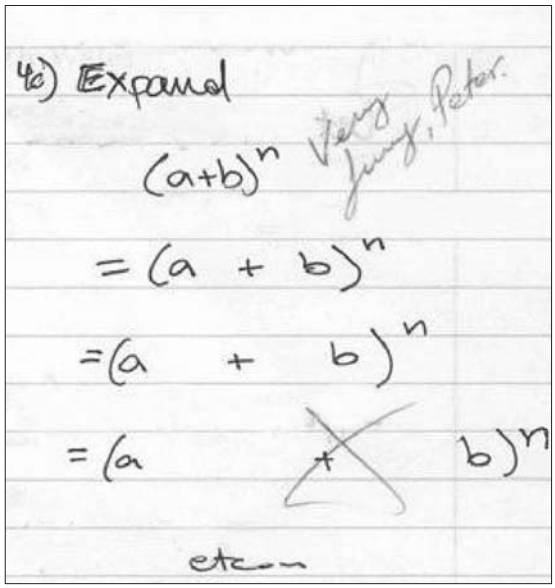
alongside everyone else? Probably, yet moments before, when seeking a *perpendicular* bisector, I had been a bit frustrated when all the pupils tried to crowd on top of one another at the midpoint of the two fixed positions! ("Isn't there anywhere else you can stand?") One minute I *want* thinking 'outside the box', the next minute it *irritates* me. Is this fair?

The first time pupils discover the 'zoom' facility on a computer graph-drawing package, I find that they nearly always try zooming in and in and in, sometimes asking about the standard form notation that appears on the axes. They are usually surprised that they can keep going and going and that the lines don't get thicker or that a graph like $y = \frac{1}{x}$ really *doesn't* seem to touch the axes, no matter how far you scroll and zoom. Sometimes they put their faces right up close to the screen as they do it, to try to see where the lines are coming from. Even when these activities end up crashing the computer, I feel that this sort of engagement is mathematical and should be accepted – welcomed, even. Saying, "Don't be silly" just doesn't feel like a mathematically sound reaction. Personally, I feel that anyone who has never tried scrolling down and down a computer spreadsheet, for instance, to see whether the cells really go on for ever, has a slightly disturbing lack of curiosity! (See also the 'overheard' on the right.)

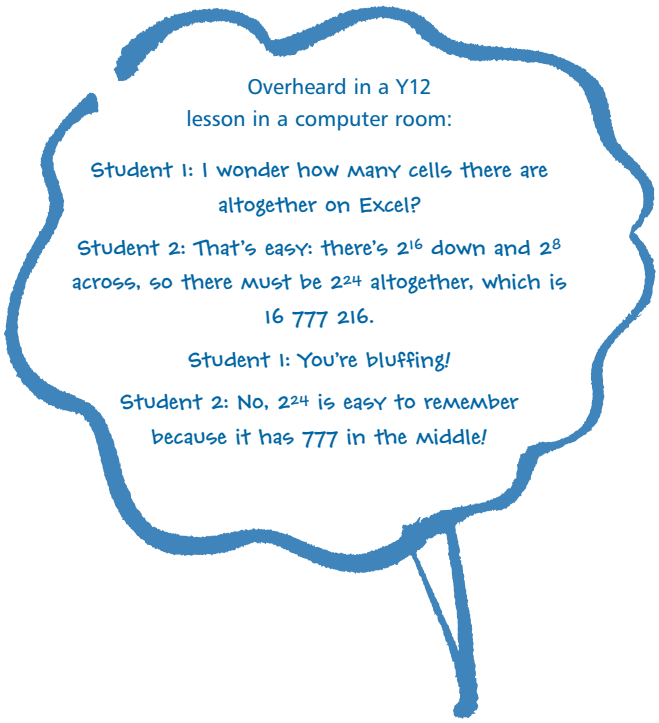
For many pupils, mathematics is not a favourite subject. Does some of that stem from the insecurity of not knowing what is 'acceptable' mathematical behaviour and what isn't? Sometimes a long complicated problem has a short simple answer, and you feel that you haven't done enough 'work' and you're going to get into trouble. Sometimes the simplest of questions leads to pages of scribbling and no easy resolution and you don't know what to do. Sometimes giving an 'off-the-wall' answer like 'pi' leads to a reward; sometimes it leads to a detention. Sometimes when you use a short-cut like seeing an enlargement of a '3-4-5' triangle or spotting a difference of two squares or solving an equation by inspection you are praised for being clever; sometimes you're criticised for being lazy and not showing enough 'working'. There has to be a way though all of this to allow and encourage pupils to 'misbehave' mathematically in ways that extend their mathematics. I don't think you can do it through lists of unbending 'classroom rules' or simplistic notions of 'time on task', and I think that the teacher's attitude and example to questioning the obvious and engaging with surprises is crucial.

Colin Foster teaches mathematics at King Henry VIII School, Coventry, and is an editor of *MT*.

An Alternative Interpretation . . .



With thanks to Bob Burn for sending this in.



OVERHEARD

Have you overheard a snippet of conversation that has amused you or offers a surprising perspective on an issue or provides an insight into your students' thinking? We would like more material for 'Overheard'
– Editors (email addresses on page 1).

The Association of Teachers of Mathematics
for mathematics educators
primary, secondary and higher

This document is copyright ATM and others. Permissions: copyright@atm.org.uk

7 Prime Industrial Park • Shaftesbury St • Derby • DE23 8YB • +44 (0) 1332 346599 • www.atm.org.uk • info@atm.org.uk

The work that went into the research, production and preparation of this document has to be supported somehow.

ATM receives its financing from only two principle sources: membership subscriptions and sales of books, software and other resources.

Membership of the ATM will help you through

Now, this bit is important - you must read this

- Six issues per year of a professional journal, which focus on the learning and teaching of maths. Ideas for the classroom, personal experiences and shared thoughts about developing learners' understanding.
- Professional development courses tailored to your needs. Agree the content with us and we do the rest.
- Easter conference, which brings together teachers interested in learning and teaching mathematics, with excellent speakers and workshops and seminars led by experienced facilitators.
- Regular e-newsletters keeping you up to date with developments in the learning and teaching of mathematics.
- Generous discounts on a wide range of publications and software.
- A network of mathematics educators around the United Kingdom to share good practice or ask advice.
- Active campaigning. The ATM campaigns at all levels towards: encouraging increased understanding and enjoyment of mathematics; encouraging increased understanding of how people learn mathematics; encouraging the sharing and evaluation of teaching and learning strategies and practices; promoting the exploration of new ideas and possibilities and initiating and contributing to discussion of and developments in mathematics education at all levels.
- Representation on national bodies helping to formulate policy in mathematics education.
- Software demonstrations by arrangement.

Personal members get the following additional benefits:

- Access to a members only part of the popular ATM website giving you access to sample materials and up to date information.
- Advice on resources, curriculum development and current research relating to mathematics education.
- Optional membership of a working group being inspired by working with other colleagues on a specific project.
- Special rates at the annual conference
- Information about current legislation relating to your job.
- Tax deductible personal subscription, making it even better value

Additional benefits

The ATM is constantly looking to improve the benefits for members. Please visit www.atm.org.uk regularly for new details.

LINK: www.atm.org.uk/join/index.html