Working without a Safety Net

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Since my background is in science, people often ask me why I teach mathematics instead. After all, as a science teacher you have the advantage of hooking learners with all their senses: flashes, bangs and smells. The science labs are certainly something that many learners at the start of secondary school (high school) are eager to explore, and when we have an open day at school it feels unfair that the mathematics department has to try to compete for excitement with the liquid nitrogen and explosions on display in the science area.

Well, there is nothing wrong with being a science teacher, but for me school science teaching is full of restrictions. Some things are too valuable to buy or destroy, such as dissolving a gold ring in aqua regia. Some things are illegal or unethical, such as passing electricity through a live human brain. Some things are too small or too large to see, such as quarks or a supernova. Some things take too long to observe in a 30-minute lesson, such as human evolution. But above all, some things are deemed to be too dangerous to allow learners to do for themselves. At best, they watch a teacher 'demonstrate'; at worst, they watch a film of it. Even when learners can try things for themselves, they have to be given permission to do so. The teacher has to approve in advance—learners are not allowed to engage in their own private experiments without prior approval and supervision. And with good reason.

I wonder whether we mathematics teachers realise how lucky we are? It may be true that "black holes are where God divided by zero", but learners can attempt to divide by zero by hand, or even on a calculator or computer (e.g., plotting $y = \frac{1}{x}$ on a graph plotter) without anything breaking. No-one gets hurt if they try to square root a negative number; no-one has to go to hospital because they misplaced a minus sign in their algebra. What I love about teaching mathematics is that learners can explore without my permission—no teacher needs to impede their thinking by fencing off areas and telling them they are too young or too inexperienced or too clumsy to go there. Sometimes learners will tell me about some mathematics that they have been thinking about at home (or in assembly) or they will show me some mathematics that they have been doing while I thought they were doing some other mathematics, and there is nothing wrong with this – in fact, it is a delight. As their teacher, I don't have to be scared about what my learners might be up to mathematically; I don't have to be a control freak in the classroom ("Get your hands off those gas taps!").

And yet learners so often are hemmed in during mathematics lessons. Why do we do that? Perhaps sometimes we lack confidence in how to do deal, either mathematically or pedagogically, with what might arise. When I taught science, children would sometimes bring in objects from home and ask me how they worked, and often I did not really know. I wanted to take them to pieces to see, but did not feel I had permission to destroy them. Unpicking mathematical ideas, on the other hand, is not going to lead to a parent complaining that I have ruined their precious gadget!

Also, teachers fear that learners will run up against mathematical constraints and get hurt *emotionally* and become discouraged and down-hearted about the subject. At all costs, teachers try to prevent problems and 'failure' and I think that this underestimates the resources that learners bring to their studies. Learners are much more likely to be discouraged by being told that things are too hard for them than by discovering for themselves something complicated.

This is one of the dangers of being an experienced teacher. You have seen it all before, you know what can go wrong, and there is a big temptation to avoid problems. You are always anticipating, and this can be claustrophobic for learners. So often, it is when difficulties arise that lessons are interesting and memorable for learners.

Suppose a less experienced mathematics teacher is just about to go and teach enlargements to her Year 8 (age 12–13) class. She comes into the mathematics staffroom and picks up a stack of squared paper. A more experienced colleague says, "Oh, I've got some good enlargement worksheets you can use if you like. I always use them and they work like a dream."

"Oh, I was just going to get them to try it on squared paper," she replies. "I was going to say they could choose a shape and a scale factor and just do the enlargement for themselves."

"Ah, yes, but what happens if you do that is they always end up with the image going off the edge of the paper. And they get frustrated and you'll be running around from one to the other, and it's much easier if you photocopy sheets where you know the image is going to fit on. Then you can tell them that if it goes off the page they've done it wrong."

This teacher has a choice: the 'easy' way or the 'hard' way. Overhearing a conversation along these lines got me thinking about how to utilise this 'problem' of images not being on the paper rather than prevent it. So I made a worksheet as well, but (I hope) of a rather different kind from the one described by this imaginary teacher (Figure 1).

> Enlarge the triangle with a scale factor of 3 using different centres of enlargement.
> Where can the centre of enlargement be so that the image lies completely on the grid? Why?

Have you ever tried this problem? What do you think the locus of possible centres of enlargement will be like? Will it be a circle around the object? Will it be rectangular because the grid is rectangular? Or will it be triangular, because that is the shape of the object? Or a kind of compromise between the two? What happens if you move the object to a different position? Or change the scale factor to 2 or something else? What if you change the shape of the object?

When I gave this to my Year 8 class, some learners clearly gained a lot of practice at making enlargements, producing far more than even the most traditional textbook would have required (Figure 2)! In the time that they

Enlargement

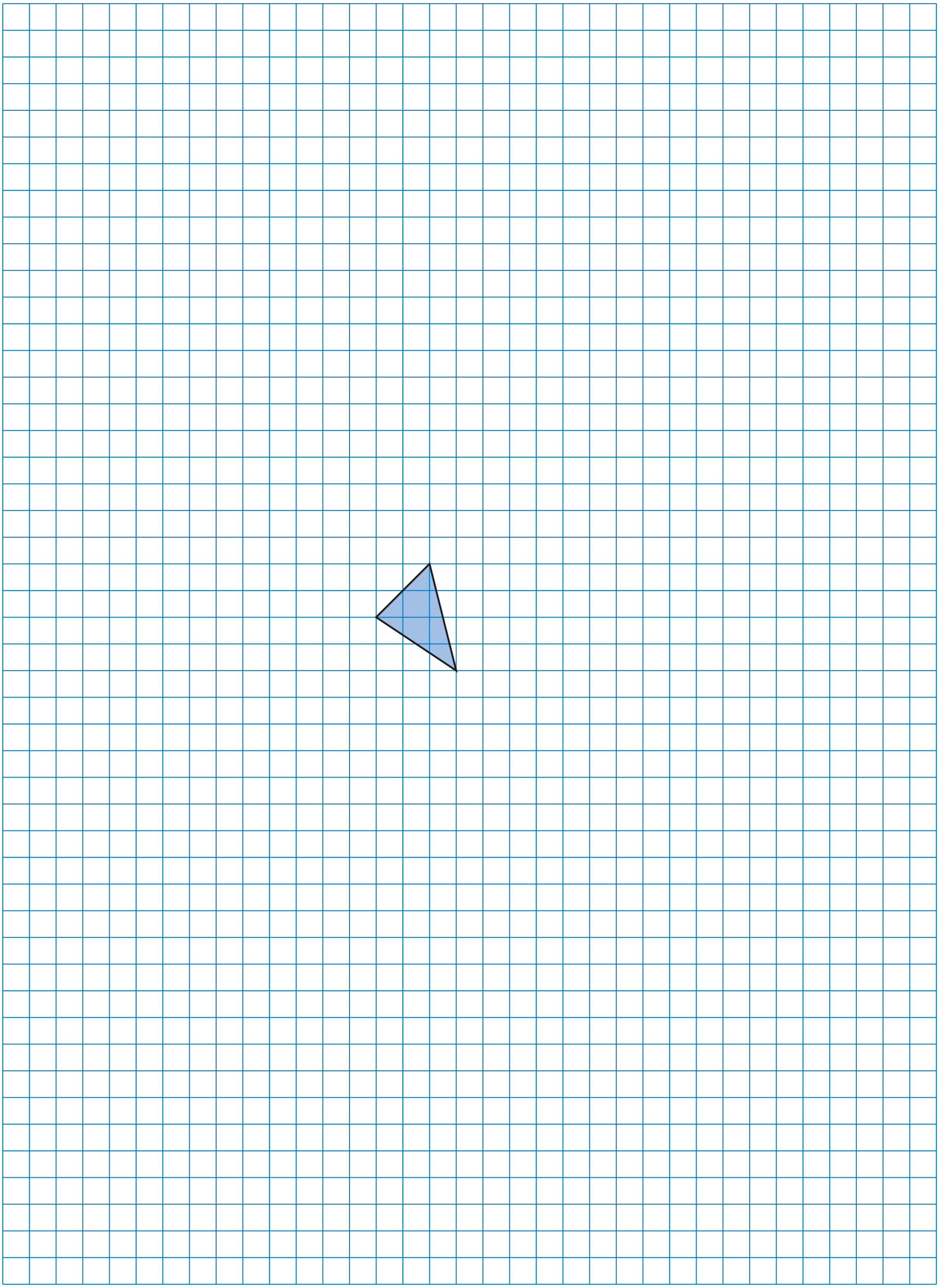

Figure 1. Enlarge the triangle with a scale factor of 3 using different centres of enlargement.
Where can the centre of enlargement be so that the image lies completely on the grid? Why?

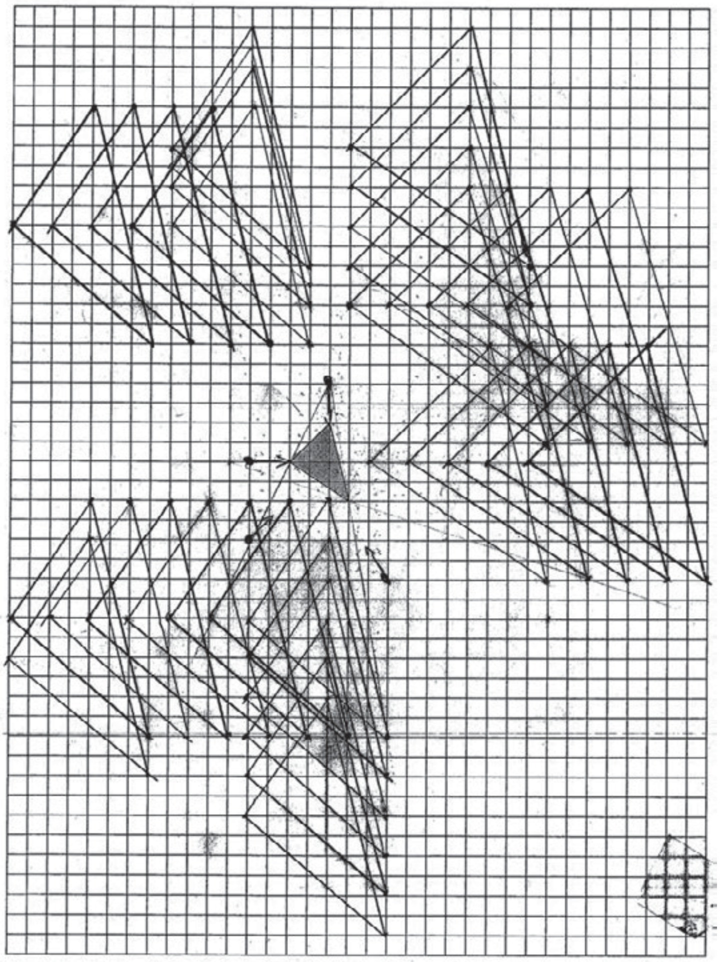

Figure 2

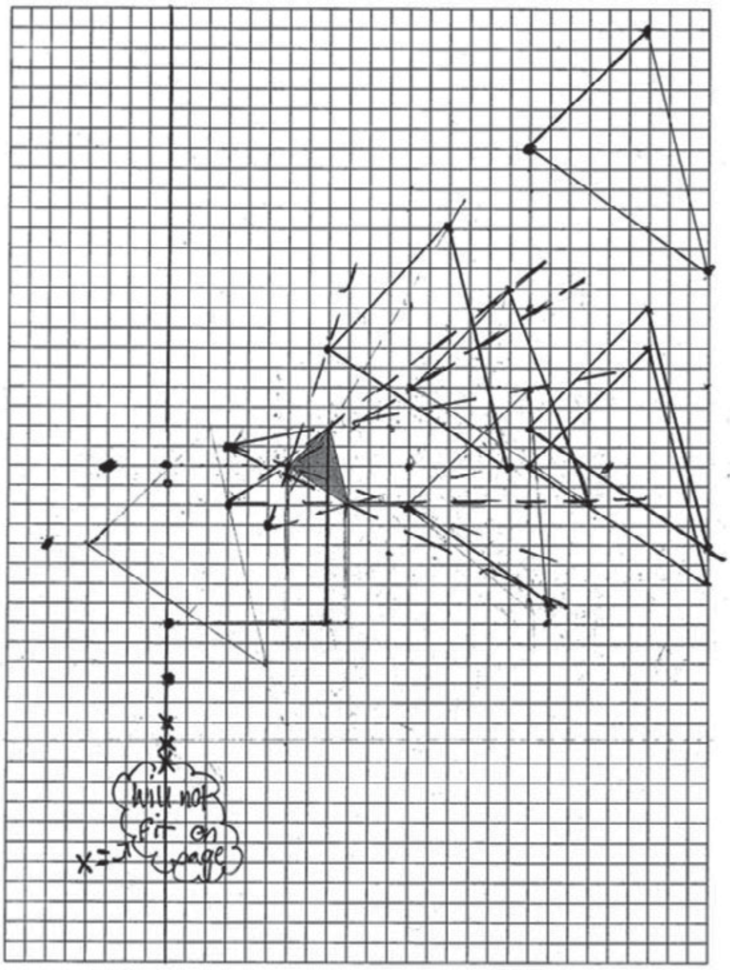

Figure 3

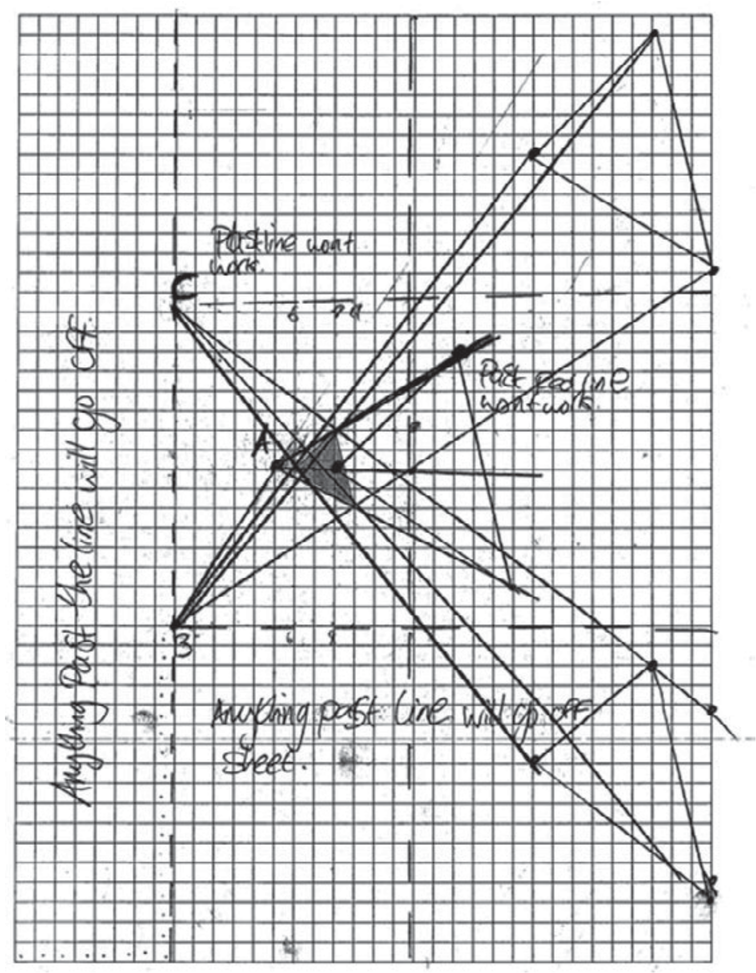

Figure 4

had, some found one boundary only (Figure 3) and several obtained a more or less complete solution (Figure 4).

I used to say that I teach mathematics because I love the power and precision of the subject—in mathematics the graphs do not have any error bars. But there is more to it than that. I love the way that learners of mathematics can take control and come to conclusions that they know are right, not because someone tells them so but because they have reasoned it out for themselves. There is an 'authority' issue, for me, in teaching most other things—a point where the learner has to say, "OK, Sir, so is that right?" and a tension in the air as the teacher ponders and eventually says, "Yes". Ideally, in science there is no human 'teacher'—the real world is the ultimate arbiter. No matter how great a scientist you may think you are, you do your experiment and then you have to wait nervously for the result to come in, which will make or break your beautiful theory. Nature has the final say; it is very humbling. In mathematics, it is thinking that has the final say, and this can be anybody's thinking—there is no hierarchy. In mathematics, it does not matter who you are.

Paradoxically, 'safe' lessons are dangerous. We miss the whole point of teaching mathematics if we adopt a policy of preventing the unexpected—we throw away one of our biggest advantages. When you feel like playing safe, it is worth asking yourself, "What's the worst thing that can happen?" Mathematical errors are not expensive; our insurance premiums are not going to rise if something interesting happens in the classroom. Let us live dangerously and be more ready to throw caution to the wind and give learners the chance to experiment and find out what doing mathematics is really like.