Proof Without Words: Integer Right Triangle Hypotenuses Without Pythagoras

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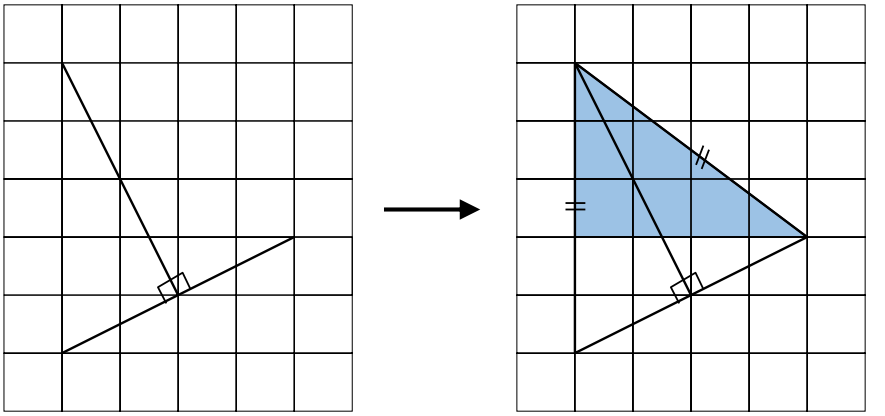
Theorem. *A right triangle with legs 3 and 4 has hypotenuse 5.*

Proof.



A referee offered the following generalization. Consider integers $m > n > 0$. Draw line segments from $(0, 0)$ to $(2mn, 2n^2)$ and from $(0, m^2 + n^2)$ to (mn, n^2); these are perpendicular. The corresponding right triangle has legs $m^2 - n^2$ and $2mn$ and hypotenuse $m^2 + n^2$. Any primitive right triangle is given by such a pair with the additional criteria that m and n are relatively prime and have opposite parity [**1**].

Summary. Without reference to the Pythagorean theorem, we show that a right triangle with legs 3 and 4 has hypotenuse 5. The figure can be modified for other integer right triangles.

Reference

1. W. Sierpiński, *Pythagorean Triangles*. Trans. A. Sharma. Yeshiva Univ., New York, 1962. Reprinted by Dover, Minneola, NY, 2003.

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