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Access to our school buildings is by a four-digit security code which is changed regularly, presenting a challenge for students (and staff) to remember the current code. When a new code is announced, I usually ask my classes to find some "mathematical" way of remembering it. This may be based on special properties possessed by the four-digit number (e.g., see www2.stetson.edu/~efriedma/numbers.html) or on connections between the digits. For example, the calculation

$$1 + (2 \times 3) = 7$$

might offer a way of remembering the code 1237.

Recently, our security code was 1682,[1] which many students remembered as

$$16 \div 8 = 2.$$

When it was time to change the number, a new code was constructed by adding 1 to each digit:

[1] This code, and those following, are the actual codes used, although all are, of course, now obsolete!

$$\begin{array}{cccc} 1 & 6 & 8 & 2 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 7 & 9 & 3 \end{array}$$

However, it was quickly noticed that this new number retained the same property; i.e.,

$$27 \div 9 = 3.$$

This seemed quite extraordinary—had the head teacher deliberately intended this, we wondered, when choosing the numbers for the first code?[2] How likely would it be for any particular number with this property to keep it when 1 is added to each digit? It is easy to construct numbers such as 5678, such that $56 \div 7 = 8$, but adding 1 to each digit produces 6789, and $67 \div 8 \neq 9$. So were there any other four-digit numbers besides 1682 which possessed this property *before and after* adding 1 to each digit?

Clearly, we can swap around the third and fourth digits, because $16 \div 2 = 8$ also, so 1628 would work just as well as 1682. So it seems sensible to restrict the third digit to being greater than or equal to the fourth, to prevent a trivial doubling of solutions. We then want

$$a \quad b \quad c \quad d$$

such that

$$10a + b = cd \quad \text{and} \quad 10(a+1) + (b+1) = (c+1)(d+1),$$

with $c \geq d$ and $c \neq 0$.

Subtracting the first equation from the second, we obtain

$$c + d = 10,$$

meaning that in the positive single-digit integers we have only five possible cases, as shown in Table 1.

Table 1.

a	b	c	d	$a+1$	$b+1$	$c+1$	$d+1$
2	5	5	5	3	6	6	6
2	4	6	4	3	5	7	5
2	1	7	3	3	2	8	4
1	6	8	2	2	7	9	3
0	9	9	1	1	10	10	2

[2] He informed me that he hadn't and that his intention in adding 1 had been to wear out different buttons on the keypads as well as to avoid a key which was broken on one of them.

The fifth of these does not work, since the larger of the two numbers requires digits greater than 10, so we see that in addition to

$$1682 \quad \leftrightarrow \quad 2793$$

there are exactly three other possibilities:

$$\begin{array}{ccc} 2173 & \leftrightarrow & 3284 \\ 2464 & \leftrightarrow & 3575 \\ 2555 & \leftrightarrow & 3666 \end{array}$$

Investigative work such as this on "real-life" mathematics can encourage students to explore patterns in numbers that they encounter in the wider world [1]. For example, students now frequently comment on numerical observations concerning the date (written in various formats), and this can provide a way of them examining various properties of numbers.

Reference

1. C. Foster, *Mathematics for Every Occasion*, Association of Teachers of Mathematics, Derby, 2009.