Questions
Pupils Ask!

How many detentions will I get?

by Colin Foster

At our school we have a 'penalty points' system where two or three points (a maximum of five) can be given for minor offences such as running down the corridor, being late for lessons or forgetting your PE kit. When you reach 15 penalty points, you get a detention. A particular pupil had forgotten his calculator for maths two days running.

Me: Well, if you haven't got your calculator tomorrow then I think I'll give you *a million* penalty points!

Jack: A million!

Me: That seems fair, don't you think?

Alex: Woah! How many detentions would *that* be?!

Jack: Er [reaching on the desk for his calculator but then remembering he doesn't have it] I don't know!

Me: [As Alex goes to lend him his, I take it] No, since you haven't bothered to bring your calculator you obviously think it's unnecessary. How can you do it without a calculator?

Jack: ... Well it's a million divided by 15 ... I can do a million divided by 10 – that's 100 000. A million divided by 20 – that's 50 000, so it must be half way between 100 000 and 50 000, so it's 75 000 detentions.

Alex: Wow! And how long would it take to do them – one a week [he has his calculator back now] – that's 1443 years, and that's not counting holidays!

Me [Looking at Jack] 75 000 is a good approximation but it's not exactly right.

Jack It is: 50 000 to 100 000 is two lots of 25 000, so it must be 75 000.

Me 12 divided by 4 is 3, 12 divided by 2 is 6, so what should 12 divided by 3 be?

[writing it down]
$$\frac{12}{4} = 3$$
$$\frac{12}{3} = ?$$
$$\frac{12}{2} = 6$$

**Jack:** 12 divided by 3 is 4.

Alex: But it should be $4\frac{1}{2}$, because that's half way between 3 and 6.

Jack: That's weird.

Alex: ... A million divided by 15 can't be exact, because you can divide it by 5 but then whatever you get you won't be able to divide that by 3, because 3 doesn't go into a million, because the digits add up to 1 ... [he now uses a calculator] so it's actually 66666.66667.

Jack: [Looking at the calculator display] That's 6 recurring, probably. Why doesn't my way work?

I couldn't see on the spur of the moment how to develop this for them, and I was needed elsewhere in the classroom, so I said something weak like "That's interesting; have a think about it" and continued to circulate round the room. I had an image of the graph $xy = 1\,000\,000$ not being a straight line, but the pupils would not be familiar with reciprocal functions. Maybe this would be a way of introducing them to inverse proportion? I remember from my own school days it striking me as odd that direct proportion was a straight line upwards but that indirect proportion was *not* a straight line downwards but a *curve* – why the asymmetry?
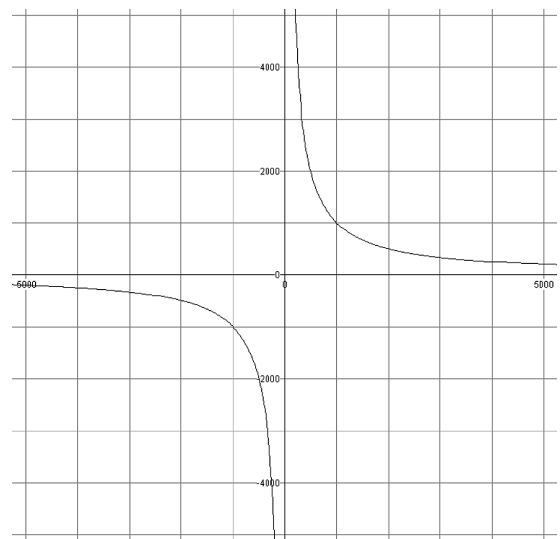
$$y \propto \frac{1}{x}$$
$$y = \frac{1\,000\,000}{x}$$

Jack had been assuming a linear decrease ($y \propto (k - x)$), or, specifically, $y = 15\,000 - 5\,000\,x$) rather than a hyperbolic one. The incident reminded me of how persistent and
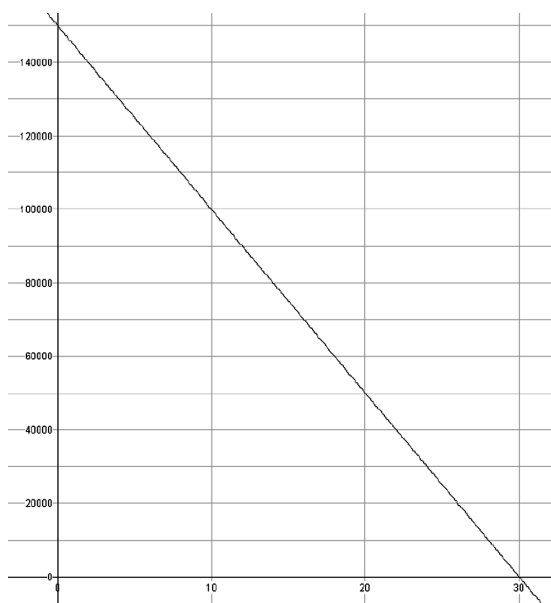
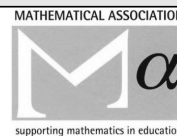$$y \propto (k - x)$$
$$y = 150\,000 - 5\,000x$$

ubiquitous are people's assumptions of linearity, and also of the way in which informal approaches to arithmetic can sometimes unexpectedly go awry. ▱

Keywords: Graphs.

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