Failing to see both sides

Colin Foster explores the benefits of being playfully awkward.

I have noticed that sometimes, in a learning situation, I will choose to be deliberately awkward, hopefully in a playful rather than a confrontational way, and pretend *not* to be able to see what the learner sees. Might this sometimes be a good strategy, and, if so, when? Our instincts as teachers tend towards being sympathetic and understanding, always trying to see things from learners' perspectives whenever possible, and surely this is a good thing. But perhaps there are times when being deliberately awkward, and purposefully *not* seeing what the learner sees can be more helpful.

A seven-year-old child was drawing on some scrap paper, which was blank on one side and had some rejected article of mine printed on the other side. She wanted to change to some paper that was blank on both sides, so she asked for some paper "with two sides".

Me: Doesn't every piece of paper have two sides?

Child: I mean *blank* on two sides!

So, she fetched some blank-on-both-sides paper.

While she was drawing, I asked:

Me: Do you think I could show you a piece of paper that has only *one* side?

After some thought, she decided no – this is impossible. So, I asked her to cut a strip of paper, and I bent it around, gave it a half twist, and taped it to make a Möbius strip (Figure 1).

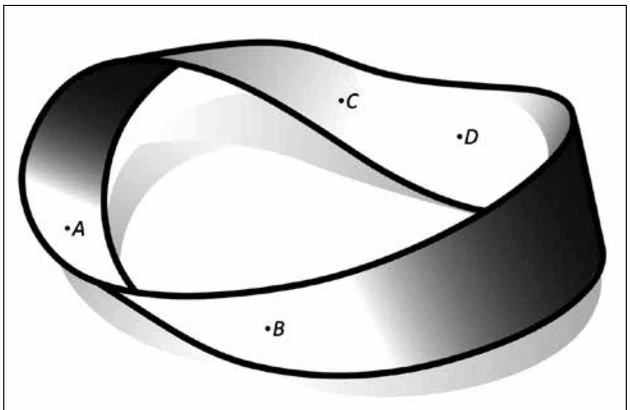

Figure 1. A Möbius strip with four points marked.

Me: This piece of paper has only one side.

Child: No it doesn't. It's got two sides. This side and this side. [She pointed at two positions like *A* and *B*, shown in Figure 1]

Of course, I can completely see her point of view here. These two points really *do* look as though they are on different sides of the paper. Möbius strips are highly counterintuitive. I could acknowledge this, and we could work on a mathematical definition of 'side' that would force us to conclude that they are, in some sense, 'on the same side'. But I find that this kind of approach tends to lead to the idea that mathematicians are strange people – they have funny ways of defining things, which indeed is sometimes true. Although common sense tells us that *A* and *B* are on different sides, there is some precise and technical way in which mathematicians prefer to consider them as being on the same side. This is strange, and we just have to get used to it. Instead of this, I wanted the child to see that the only reasonable, common-sense position to take is that these points are on the same side. It takes some thought, but to conclude otherwise would lead to some very strange consequences. There is nothing here about mathematics being perverse.

So, I chose to be deliberately awkward and to feign puzzlement at her comment. Although her response was totally reasonable, and exactly what I would have expected her to say, I chose 'not to see both sides'. I chose not to be understanding, and I decided to (role) play the part of being unable to see her point of view.

Me: [Looking puzzled] You just pointed to the same side twice.

Child: No, I didn't. *This* side and *this* side. [She repeated pointing to *A* then *B*.]

Me: That's like saying '*this* side and *this* side'. [I pointed to two positions like *C* and *D*, shown in Figure 1.]

Topologically, these pairs of positions are equivalent – mine just happen to be a bit closer together. But, of course, they do look completely different.

Child: That's the *same* side!

Me: Every reason why you think these [*C* and *D*] are on the same side applies to your two points [*A* and *B*] just as well. Why do you think these [*C* and *D*] are on the same side?

Child: Because you can just go ... [she slides her finger from *C* to *D*].

Me: You can do that with these [*A* and *B*].

She traced a pen all the way round the strip from *A* to *B*, seeing that she can do this without crossing any edges. It looked as though getting from *A* to *B* involved crossing an edge, whereas getting from *C* to *D* did not. She thought that you can get from *C* to *D* by crossing *two* edges (or the same edge twice), and she discovered that you can also get from *A* to *B* by crossing *two* edges, if you go all the way round the loop. This led her to notice that the Möbius strip also has *only one edge*, colouring the edge with a felt-tip pen all the way round and back to where she started.

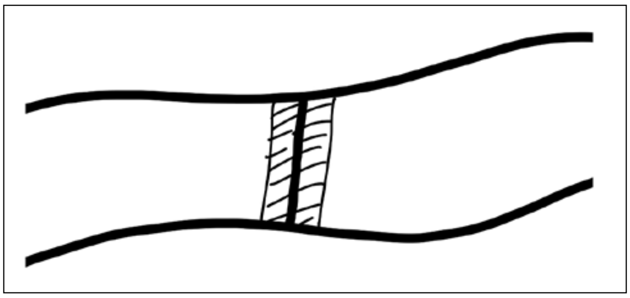

Figure 2. The place where the two ends of the strip meet and are taped together.

The idea of not going over an edge is important, but this led to a 'sticky' issue when she began to think about the spot where we had taped the two ends of the strip together (Figure 2).

Child: You cross the edge here!

This observation seemed to threaten the whole discussion. Like someone exposing how a magic trick was done, here she had uncovered the place where all the action was happening – *this* is where we switch to the other side of the paper.

Child: If the paper was coloured on one side, this is where you would move to the other colour, on the back of the paper.

To me, this join was of no importance at all, although it seemed critical to her. For me, you could smooth out this join as much as you wished, and this position on the strip was no more significant than any other position. But to her this was the crucial spot where everything turned.

Me: Imagine the strip was made out of plasticine, rather than paper. Then you could smooth it down and you could never find out the place where someone had made the join.

Child: But plasticine doesn't really have definite sides, like paper does. You can always squish it into something where the sides get mashed together.

We were into the 'rules of the game'. What counts as a shape, an object, a side, an edge? It was a very engaging discussion for us both. And I think that sometimes being awkward and deliberately 'failing to see the other side' may be helpful for enabling someone else to see something and articulate it. Are there other situations in which 'awkwardness' can be a productive pedagogical strategy?

For me this was an interesting episode to reflect on, because it 'felt right' and also 'felt wrong' at the same time, which made me doubt my approach. I have acted in similar ways when working with number bases other than 10. We might be 'in' base 8, for example, but a child will say 'nine', which it is very easy to fall into doing. I will feign lack of comprehension: "Nine? What is this thing called 'nine'? Digits only go up to seven." In that situation it seems like an amiable kind of way to react to deviations from a rule. As in the case of the Möbius strip, these approaches seem to be a kind of attempt to block one way of seeing, so as to create space for another.

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