Improving educational design by comparing alternatives

Colin Foster suggests that educational design might be enhanced by designers creating two versions and comparing their similarities and differences.

At Loughborough University, we are currently working on designing the LUMEN (Loughborough University Mathematics Education Network) Curriculum https://www.lboro.ac.uk/services/lumen/curriculum/. This will be a completely free set of mathematics resources for ages 11 – 14, with accompanying guidance for teachers (Foster *et al.*, 2021). I have been thinking about how we might elevate the quality of what we produce. When I reflect on the typical design process, too often it can be roughly characterised as:

1. Think of an idea.
2. Produce some materials.
3. Show them to colleagues for feedback: "It looks great!"
4. Show them to teachers for feedback: "It looks great!"
5. Ask some teachers to try it with some classes: "It went really well!"
6. Make minor adjustments to accommodate any small bits of critical feedback.
7. Publish it.

People tend to be polite, especially when you are giving them free things. After all, if they do not like them then they can just not use them. I generally find that people are not very inclined to be critical. Teachers, especially, tend to be positive, constructive, supportive people – it is in our training – and especially so if they know that it is something that you have spent a lot of time working on and are heavily invested in. No amount of saying, "I really want you to be critical." seems to help. So, I think that having teachers say that some resources are 'great' or that a lesson 'went really well' does not really tell the designer very much or help them to improve the things that they are producing.

One response to this is to say that asking teachers what they think is the wrong methodology. What we really care about is not so much teachers' opinions about resources but whether learners benefit from them. Every teacher knows that they can often be surprised by what happens in the classroom. So, we should really be carrying out randomised controlled trials to see whether learners' learning improves more when using one design than it does when using some other alternative. However, the desired outcomes of learning can be very hard to pin down. Different, competing, resources, even if on the same topic and with ostensibly the same broad objectives, may really be trying to do quite different, subtle things. Establishing a test that is fair to both is really difficult. What the teacher and learners do with the resource may matter much more than the resource itself, and trying to tie down how the resource is used by providing detailed teacher guidance may just inhibit teachers from maximising its potential and making productive adaptations.

In addition to this, benefits from a resource may often be delayed or may depend on how it relates to learners' other experiences. If you are trying to design a curriculum that fits together coherently, prioritising connections across different ideas, then sometimes you might be willing to do things in more difficult ways, for a greater payoff later. This means that it may not make sense to trial components of such a curriculum, because these would not be expected to show their benefits in isolation but only when implemented across the whole. All of this means that it can be very challenging to conduct informative trials of resources that will actually help designers to know if their designs are any good, and, importantly, enable them to improve their work.

Comparing alternatives

One way to address these problems could be to avoid asking their designer colleagues or teachers to comment on a single resource, and instead offer them, 'two for the price of one'. Present two different resources that are designed to do, in some sense, 'the same thing', and ask, "What is the same and what is different about them?" Neither of these alternative resources should be a straw person; that is, something that the designer put little effort into, does not like very much and would probably avoid using themselves. There should not appear to be a right answer to which one is better. Both options must be realistic alternatives, both designed carefully to a high standard by a designer who believes in it. If the designer has any preference for one over the other, this should not be apparent. The point of comparing alternatives is that the designer is genuinely unsure about the advantages and disadvantages of each and wants to have some input into thinking about it more carefully.

The intention would be that if either of the two resources were presented on its own, teachers or other designers would be likely to say, “It’s great!” So, instead of asking whether people like the resources, we ask them to compare and contrast them by describing in detail what they see as ‘the same and different’ about them. What different opportunities could each provide for learners? What difficulties might there be in using each of them? How well or how badly could each of them mesh with other related ideas and resources? How would you need to adapt each of them to make it work for you and your learners in practice?

The methodology of ‘comparative judgment’ (Thurstone, 1927) exploits the fact that human beings find relative judgments much easier to make than absolute judgments. For example, if someone is asked to hold a cricket ball in their hand and estimate its mass (an absolute judgment), different people tend to give wildly different estimates. But, if instead they are asked to hold a cricket ball in one hand and a tennis ball in the other, they can quite easily say which one is heavier (a relative judgment). In the case of mathematics resources, giving people two resources to compare may make evaluation easier and comments more insightful because each has the other as a reference point.

Prompts could include:

- Both resources are ...
- Resource 1 is more ... than resource 2.
- Resource 1 is less ... than resource 2.
- Neither resource is ...
- I prefer ... about resource 1 because ...
- I would prefer to use resource 1 because ...

Perhaps a comparative methodology considering pairs of resources could provide more nuanced and insightful comments than evaluation of a single resource by itself.

Of course, this process seems costly in terms of design time: the designer’s job just doubled. Instead of just writing a task or lesson they now have to write a pair of contrasting ones. But this might not always necessarily be as much work as it might seem. Sometimes, merely changing the order of activities (for example, discussing-then-exploring versus exploring-then-discussing) could be an easy variation to make that could profoundly affect what happens. In other situations, instead of getting stuck in choice paralysis over which direction to go in, working up both options and comparing them might even take less time and could be helpful. Nevertheless, even when twice as much time is needed to develop two versions, perhaps this effort is likely to be creatively productive. When designing a single resource, the danger is that the designer simply goes for the first thing that comes into their head, perhaps thinking, “How have I done this before?” and then uncritically following that path. As they progress further into the design process, they become increasingly invested in what they have done so far, and there is a big sunk-cost disincentive for giving up and starting all over again in a completely different direction. I can certainly recognise that in myself. If some early feedback suggests that this is ‘good enough’, then they may be likely to stop there and feel pleased with themselves that they have done a good job. So, I suspect that we could elevate our design work by having a discipline of ‘Do it twice; one design is never enough’. This forces the designer to stop and ask, “If I weren’t going to do it the way I just have, then how else could I do it?” In my experience, I often prefer the second idea I have to the first.

Directed numbers

What might this kind of process look like in practice? Here I will consider a sequence of lessons focused on introducing addition and subtraction of directed (positive and negative) numbers. Below are two (I think) very different approaches to doing that. I think there are nice features to both of these approaches, but I think they are very different and could lead to quite different perceptions of the mathematics. I would seem to me that it cannot be the case that it makes no difference which one you use. To explore this further, the question posed is: “What is the same and what is different about them?”

For convenience, I present each approach on pages 17 – 18 as a list of questions, but this is of course not intended to be a worksheet that would simply be handed out to a learner. The questions are intended to indicate the kind of sequenced approach that a teacher could take. Both approaches could easily involve physical manipulatives – counters for the first (whether lined up or in more of a scattered arrangement) and hops along physical or virtual number lines for the second; number lines that could be positioned either horizontally or vertically. The intention is that the teacher would operate in this general kind of way, but not stick rigidly to a scripted sequence of worded questions.

Conclusion

Whatever you think of these two approaches, they seem to me to be really different in important ways, and certainly not equivalent and interchangeable. I do not think it can be sufficient to say that both approaches are fine and just flip a coin to decide which to use. It is hard to believe that it would make no difference which approach a learner experienced. Certainly, which model learners use can dramatically affect the relative difficulty of different questions. For example, 'taking away a negative' in a case such as (–5) – (–2) may be very simple in the opposite-charges approach (removing two 'negatives') but much harder in the number-line approach, but the reverse may be true for something like 5 – (–2) (Foster, 2020).

You might question whether they are really distinct models. Could they not somehow be merged, perhaps by lining up the positive charges against a number line? I cannot really see how to do this clearly for cases such as subtracting a negative charge. For me, they do seem to be quite distinct approaches – I cannot really think in both of these ways simultaneously – I have to choose.

Another way to try to avoid making a decision is to claim that one approach will probably 'work' better for some learners (or teachers) and the other approach for others. For me, I worry that this has a kind of 'learning styles' feel about it, now that it is widely accepted that teaching students according to their preferred learning styles does not improve their learning. If learners have previously met approaches similar to one of these before, then this could certainly be a factor, but otherwise it is not obvious why different learners should be expected to have different needs when it comes to learning negative numbers. It feels nice to sit on the fence, but it seems to me more likely that one of these is probably preferable for most learners when first meeting negative numbers.

Perhaps, ultimately, we might want every learner to experience both approaches. To the mathematics teacher, this might seem the best of both worlds – each lesson offers something different, and the richest experience is to have both. But, for the learner, it may be that experiencing both approaches, especially if in quick succession, may just be confusing, rather than illuminating. Our 'curse of knowledge' makes us think that seeing something in lots of different ways makes it clearer, because it does for us. But when you are beginning to learn something new it takes a lot of headspace to think in just *one* of these ways, and trying to see something from many perspectives at the start may just be overwhelming. Using multiple representations is costly (Foster, 2022).

A teacher might choose to approach this without a clear idea of the model that they want to prioritise, desiring to be 'neutral', because they want to draw on whatever learners might offer. This may sound like the kindest and most open attitude to take, but the result may be that several different (perhaps conflicting) models and representations quickly end up appearing on the board. This may look very rich to an observer, but to the learners it may just be overwhelming, and they may leave more confused than they started. Perhaps, if the objective is to present both approaches, the ideal could be to space them some distance apart (one in primary school, the other in secondary school?), so that there is time to get to grips with one way of thinking before later contrasting it with another. And, if this is the solution proposed, we still do not get away from having to make a decision: the question then becomes which approach you would use first, and why.

The main challenge of educational design is often not about distinguishing between good ideas and bad ideas, but is trying to discriminate between good ideas and other good ideas; especially if those different good ideas seem, in some ways, to be incompatible with one other. One approach may not necessarily be any better in an absolute sense than another, but it might fit better with other aspects of the curriculum. In the case of directed numbers, I do think that one of these approaches leads on to non-integers and multiplication and division of directed numbers much better than the other one does, and for me that makes it in some sense, the winner. Given the ways in which we are trying to prioritise the number line across our resources (Foster, 2022), it will be no surprise that that is the approach that for me wins out here. But there may be other ways of considering this that lead to the opposite conclusion. And, whatever the eventual decision, we always still need to think about what we *lose* by making any design decision, and how that loss might be compensated for through bringing in other tasks at other places in the curriculum that give learners an opportunity to appreciate different perspectives on the concept. This can be challenging because it may entail attempting to temporarily 'shelve' what was learned previously, to create space and openness for the new approach, before later being able to bring the two together and appreciate their connections.

So, what for you is the same and different about these two approaches? Or do you perhaps have a third or a fourth approach? And do you have any strong feelings about any of them? If so, why?

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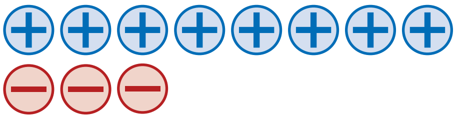
Opposite-Charges Approach

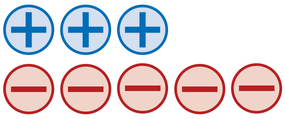
Kayla has some positive and negative counters:



How much do you think Kayla has **in total**? Why?

Can you make the same total using a **different total number** of counters?
Can you make the same total using **15 counters**?
Can you make the same total using **16 counters**?
Why / why not?

Lil also has some positive and negative counters:

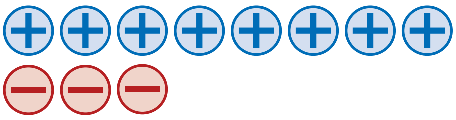


How much do you think Lil has **in total**? Why?

Can you make the same total using a **different total number** of counters?
Can you make the same total using **15 counters**?
Can you make the same total using **16 counters**?
Why / why not?

How much do Kayla and Lil have **altogether**? Why?
What do you think is the quickest way to work this out?

Kayla wants to use her counters to work out (+5) – (–2).
How would you read this calculation?



She says, "I will need to **take away** two of the negative counters".

What answer does Kayla get for (+5) – (–2)?

How could she use her counters to work out (+5) – (–3)?

What counters would Kayla need if she wanted to work out (+5) – (–4)?
What counters would Kayla need if she wanted to work out (–5) – (+2)?
What counters would Kayla need if she wanted to work out (–5) – (+3)?
What counters would Kayla need if she wanted to work out (–5) – (+4)?

Try to explain to your partner how adding and subtracting positive and negative numbers works.

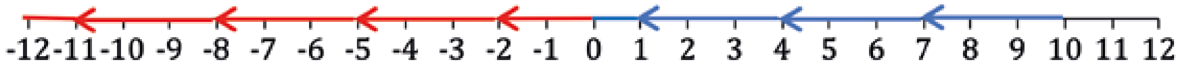
Number-Line Approach

Kayla is counting down in 3s along a number line.
She starts at 10.

Ten, seven, four, one, …

What comes after 1?

If Kayla wants an answer, she has to extend the number line to the left, by using **negative numbers**.



Now Kayla can carry on counting backwards in 3s:
Ten, seven, four, one, *negative* two, *negative* five, *negative* 8, *negative* 11, …

Without negative numbers, Kayla would have to stop at 1.
With negative numbers, how far can she carry on counting down? Why?

When might it be useful to continue counting down below zero and into negative numbers? Try to think of some examples.

Kayla writes these calculations to show the counting down in 3s:

10	–	3	=	7
7	–	3	=	4
4	–	3	=	1
1	–	3	=	–2
(–2)	–	3	=	–5
(–5)	–	3	=	………..
………..	–	3	=	………..

How would you read these statements?

What numbers go in the gaps? Why?
Continue for a few more lines.

What would happen if Kayla counted **up** in 3s? Why?
What would happen if Kayla counted **up** in **negative** 3s? Why?
What would happen if Kayla counted **down** in **negative** 3s? Why?
How would you write these as calculations?

Try to explain to your partner how adding and subtracting positive and negative numbers works.

References

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