# Getting Multiplication the Right Way Round

by **Colin Foster**

If you want to start an argument among a group of mathematics teachers, just show them Figure 1 and ask them whether it represents $3 \times 4$ or $4 \times 3$. Retire to a safe distance, put your feet up, and watch the fireworks!

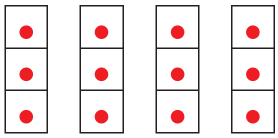

**Figure 1:** Is it $3 \times 4$ or $4 \times 3$?

In the figure, we have a group of three (the multiplicand) which occurs four times (the multiplier) – everyone agrees about this. The disagreement is over whether to write it with the multiplicand or the multiplier first. If we are inclined to read $4 \times 3$ as "four-times three", because the three comes "four times", or as "four lots of three", then $4 \times 3$ seems right, in 'multiplier × multiplicand' order. However, if, instead, you say "multiplied by" (or "timesed by"), then "three timesed by four" as $3 \times 4$ seems to fit better, with the multiplicand first (see Anghileri, 1989). Is there any way to resolve this?

One response is to say that it doesn't matter – multiplication is commutative, so there isn't a 'right' way round, and the whole discussion is a waste of time. The important thing is that students know that $3 \times 4 = 4 \times 3$. But, it is perhaps a bit tricky to show convincingly this equality of expressions if we can't decide what *either* expression actually means! And, although the two expressions are of the same value, it is too simplistic to say, "They're the same thing". Three vehicles with four wheels on each is not 'the same' as four vehicles with three wheels on each; it's the same total number of wheels, but nothing else is the same.

The dominant practice seems to differ from country to country. Watanabe (2003) found that Japanese textbooks emphasised the distinction between the multiplicand and the multiplier, always writing the *multiplicand* first, whereas in US textbooks the *multiplier* is usually written first – but commutativity is also stressed earlier, and there is less attention to the distinction (See Note). Watanabe suggested that the difference between Japanese and US practice may derive from the "4-times" language, which is common in English, whereas in Japanese the language of multiplication corresponds more naturally with having the multiplicand first.

It is easy to be agnostic about things like this until you have to design some classroom materials, and then you have to make choices – you can't sit on the fence. It seems unlikely that varying things like this haphazardly and without thought will be optimal. If consistency of some kind is desirable, then one way to proceed is to look for the pros and cons of doing it each way, so that any decision one way or the other can be made in as informed a way as possible.

In favour of putting the multiplier first, we have the "lots of" language, as in "4 lots of 3" as $4 \times 3$. This corresponds very naturally with situations such as collecting like terms:

$a + a + a + a = 4a$

$3 + 3 + 3 + 3 = 4 \times 3$

and it is natural to write the 4 first here, since we have a convention for writing $4a$ in algebra, and not $a4$.

On the other hand, in favour of putting the multiplicand first, we have the "multiplied by" language, as in "3 multiplied by 4", which we interpret as $3 \times 4 = 3 + 3 + 3 + 3$. This also corresponds with conventions on proportionality relations, where we connect two variables $x$ and $y$ by writing $y = mx$, where $m$ can be viewed as the fixed multiplicand while $x$ varies as the multiplier. Multiplicand-first also corresponds with reciting tables in the form:

$3 \times 0, 3 \times 1, 3 \times 2, 3 \times 3, 3 \times 4, \ldots$

and referring to these products as the "3 times table" then makes sense, since they all begin "3 times…" something. On balance, I feel pushed towards the multiplicand-first position, but the arguments for this don't seem overwhelming.

Of course, it may be that consistency is overrated – as Ralph Waldo Emerson put it, "A foolish consistency is the hobgoblin of little minds" – and we should just value students being flexible and being able to see a product in either of two ways?

## Note

The language of 'factor', which can be applied to either the multiplicand or the multiplier, has tended to take over in the West, and rendered this discussion moot, given that both orders can be defended mathematically (see Cunningham, 1965).

## References


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