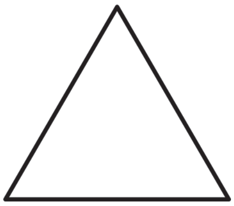
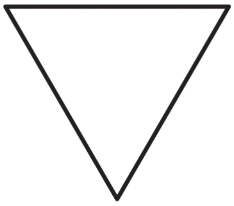
Giving Everything a Twist

by Colin Foster

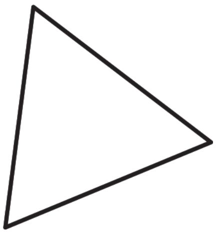
I was listening recently to some mathematics teachers talking about how presenting geometrical shapes in their standard, conventional orientations leads pupils into misconceptions (see Monaghan, 2000). If, every time a student sees a drawing of an equilateral triangle, it has a horizontal base at the bottom, like this:



then pupils may assume that this unnecessary feature is essential, and therefore fail to recognize something like:

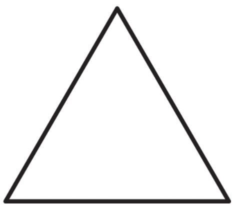


let alone

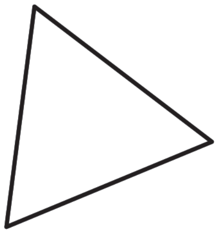


as also an equilateral triangle. The takeaway message from the conversation was that we should present shapes to students in as many different orientations as possible – preferably infinitely many, by rotating a physical (or dynamic geometry) shape continuously – in order to prevent this misconception from developing. If you flick through the pages of a typical mathematics textbook, you will see that standard orientations are overwhelmingly the norm (hence the reason we call them 'standard'), so these books are partly to blame for this problem.

I am not sure that this is quite right. If you asked me, out of the blue, to draw an equilateral triangle, I think, unless I was trying to be clever, I would be much more likely to draw



than



Does that mean that I am suffering from this 'misconception' too? Or, at least, that I am guilty of perpetuating it? Does my preference mean that I am stereotyping all equilateral triangles as *having* to have horizontal bases at the bottom? I don't think that it does mean that I am doing that. If asked to draw or imagine an equilateral triangle, I don't see why it is necessarily better to summon up one at a 'random' orientation (a 'general' equilateral triangle?), rather than a canonical, stereotypical one, with a horizontal base at the bottom. If you asked me to imagine a *person*, I would probably imagine one standing up, with their feet on the ground. This doesn't mean that I think that people *always* stand up, or that they *should* always stand up, or that they are not a person if they are standing on their head. I am just choosing a conventional, typical orientation for convenience. Does this do any harm?

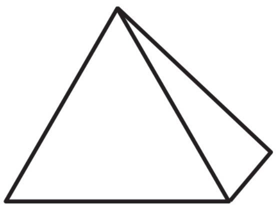
I have been thinking about things like this because, here in the Mathematics Education Centre at Loughborough, we are beginning to embark on designing a complete, free, research-informed set of curriculum resources for Key Stage 3 (ages 11-14) mathematics, and we are starting the process by trying to establish some design principles for this project (see Foster, Francome, Hewitt & Shore, 2021). We want to make decisions that are as good as they can possibly be, and consistently implement them across topics and ages. So, one possible question is: Should we make all our shapes wonky? Should this be one of our 'unique selling points' (*The Wonky Mathematics Curriculum*)? If we were to ban things like equilateral triangles in conventional orientations throughout our materials, would we be doing the students a favour – breaking down a highly prevalent misconception about how an equilateral triangle 'ought to be'? Or would we just be giving everyone a headache, with all these weird, tilted shapes, poised higgledy-piggledy all over the pages?

One possibility is that maybe we should just do this for a little while, when first introducing the idea of what something like an equilateral triangle is, and, once we've established that they can be in any orientation, we can calm down and stop worrying about this from then on?

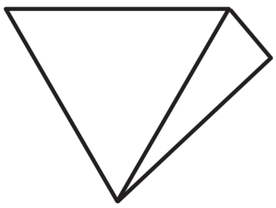
However, that approach seems to risk that valuable awareness fading away, and being swamped by the subsequent more conventional experiences. Students quickly forget about the lessons with shapes in all those funny orientations, and gradually settle into the idea that only one orientation is 'correct' – the one they (now) always see. Things tend to fade unless we keep revisiting them, so, if this is something that matters, then maybe we should keep on doing it? Or at least repeatedly interleave wonky shapes at greater and greater intervals?

One of the teachers I was listening to was complaining that when they asked students what this shape was:

the students had replied, "An upside-down triangle". There was much laughter ridiculing the idea that 'a triangle stops being at triangle when you turn it upside down – what does it become instead – a *non*-triangle?' – how ridiculous! But my response to this was to think about 3-D shapes, such as a square-based pyramid. The image that instantly comes to my mind is:



rather than, say:



More or less all the square-based pyramids I have ever seen, especially the Egyptian ones, have been what I would call 'the right way up'; so much so, that if I wanted to refer to the second one I would be very tempted to call it 'an upside-down pyramid' (we even have the name 'inverted pyramid'). If I were trying to describe this pyramid, and failed to mention the fact that it appears to be balanced precariously on its apex, that would seem to be leaving out some rather important information, surely? The fact that I call it 'an upside-down *pyramid*' suggests, I think, that I am clear that it really *is* a pyramid, just in an unusual orientation. By calling it 'upside down' I am not intending to denigrate it. The argument seems stronger with 3-D shapes that have very natural, stable orientations due to gravity. So, are 3-D shapes an exception to the general rule?

Thinking about this has left me wondering about the aesthetics of mathematics. I really don't want to go through all of the shapes that appear throughout all of our resources, twisting each one a few degrees one way or the other so that they look crooked, but should I? I think that would make our resources look quite ugly, and the extra cognitive load needed for the mental rotation for the reader to twist them back (Prime & Jolicoeur, 2010) seems an unnecessary distraction. But, unless we do this, are we guilty of building up a misconception for students? Perhaps there is a broader point about fixed views of things ("A triangle must be like this"), and subverting those views seems to be often a helpful thing to do in cultivating flexibility. Sometimes, when trying to solve a tricky geometry problem, rotating the diagram by turning the paper helps me to see something that I hadn't previously noticed. Maybe I remember some relationships in standard orientations, and find it easier to spot them if I can turn the diagram to match. This seems like a useful problem-solving heuristic for geometry (*"If you don't see anything useful, try turning the diagram a bit"*), but I am unconvinced that twisting all our diagrams in lessons is necessarily going to help pupils to do this.

So, perhaps we will compromise, and throw in the odd wonky shape now and then, but not feel that we must do this relentlessly. Perhaps this thinking will attune us to 'orientation' as an issue, and we will look for opportunities to use differing orientations purposefully in mathematical problems in varied ways throughout.

References

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