Questions Pupils Ask

Does a remote-controlled toy car violate Newton's laws of motion?

By **Colin Foster**

This might sound more like a physics question than a mathematics question, but I have found that some version of this question often comes up when teaching mechanics as part of A-level mathematics. I am often surprised how far students can get, successfully answering questions out of the textbook, and in examinations, with still a quite flaky understanding of Newton's three laws of motion.

The most prominent of Newton's laws of motion for students is the second law because this is the one that can be encapsulated within a nice formula: "Force equals mass times acceleration" (Note 1). We are forever solving problems by saying things like, "Applying N2L to the car in the positive x direction, …", followed by writing down one or more equations, which we go on to solve to find a force, a mass or an acceleration. But what about the other two laws? I have heard students ask why we 'never use them', and I think that lack of understanding of those laws lies behind much student confusion.

The first law of motion

It is easy to dismiss the first law as merely a special case of the second law, in which the resultant force happens to be zero, and therefore the acceleration also has to be zero. This means that the object is either stationary or moves with constant velocity (constant speed in a fixed direction). But I think that the first law is more than this. We can think of the first law as *defining* what a force *is*: something that causes an object to accelerate. And then the second law quantifies this by revealing the relationship between force and acceleration to be a direct proportionality. Newton's second law defines *inertial mass* as the constant of proportionality that determines *how much* force you need in order to obtain one unit of acceleration. But I think that uncertainty over what the first law is saying lies behind confusions, such as in the student's question about the remote-controlled car.

According to Newton's first law: *An object will maintain constant velocity unless it experiences a resultant external force*. This means that an object that is at rest will remain at rest, and one that is moving will continue to move at a constant speed in the same direction, unless an overall unbalanced external force acts on it. The 'external' part of this statement sometimes seems to confuse students. The student imagines a remote-controlled toy car sitting stationary on the floor. Suddenly, some distance away, someone pushes a button on the remote control, and the car moves forwards. No external force seems to be acting on the car – it's surely an *internal* force from the battery inside the car that makes it go!

As with most of my 'Questions Pupils Ask', if no student happens to ask a question like this, then I will usually pose it as a question 'from a student in another class', and ask them how they would respond. I find that lots of red herrings come up. Perhaps Newton was unaware of such possibilities as this, as he lived before the era of electronic children's toys? We can hardly blame Newton for not knowing about cars being controlled by radio waves! Students sometimes suggest that there must be an external force travelling through the ether – somehow the radio signal carries 'a force' of some unspecified kind: "Without that 'radio wave force', it wouldn't move, so that must be the external force."

To me, the distinction between a remote-controlled car and an ordinary one is immaterial here. In our thought experiment, we could scale up the car to full size and sit ourselves in the driving seat, and use the remote-control *from there*. This effectively turns the toy car into an ordinary car, because we might as well drive it normally, and dispense with the remote control. Surely someone starting the engine of an ordinary car and driving off poses exactly equivalent concerns to the radio-controlled scenario? We could imagine tinted windows if we didn't want to know whether there was a person inside the car or not. Where is the external force coming from now?

In this kind of discussion, students tend to look in the wrong place for the external force. They seem to think that the presence of the conscious being, whether inside the car or holding the remote control, is the key factor. They seem to equate 'external' with 'intentional' (Note 2), but this is a distraction. We could program the car on a timer, so that the engine would start and it would move off with no one inside it – and no person involved at all. And I try to move them towards the source of the external force by asking 'what would happen if the road were covered in very slippery ice'?

Fig. 1: The ground obligingly exerts an external, frictional force on the car that pushes it forwards (similarly at all four wheels, even if the car isn't '4-wheel drive').

Now, no matter what the person inside the car does, the car won't move forwards – the wheels will just spin. A similar scenario would be a car in the workshop for repairs, raised up in the air on a vehicle lift. Thinking outside the box here means thinking outside the *car* to the ground that it is running on, and the friction that the ground provides to the tyres (Figure 1). Friction is the most underrated force in mechanics! To move, the car needs the ground to push it forwards. I find that, even for students who have been happily using $F = ma$ successfully for some time, this can be a lightbulb moment.

But surely if the arrows in Figure 1 are representing *friction* then they are pointing the wrong way? The car is driving *forwards*, and doesn't friction always *oppose* motion? One way to address this is to start by imagining the situation on ice, where the brakes have locked the wheels, so they aren't turning, and the car is just skidding along the ice. In that situation, the arrows *would* point in the *opposite* direction (Figure 2). What little friction the ice is offering is (albeit slowly) *decelerating* the car.

Fig. 2: On a very icy surface, the wheels are locked and sliding forwards, and friction now slows the car down (similarly at all four wheels).

In Figure 2, the wheels are not behaving as *wheels*, because they aren't turning, and the car is analogous to a sledge. However, when a car is being driven normally, the wheels *will* be turning, and if the car is moving to the left then the wheels will be turning *anticlockwise*. If we zoom in on what is happening at the ground (Figure 3), then we can see that, at the point of contact with the ground, the wheel must be pushing on the ground to the *right*. So, the frictional force *will* oppose that *relative* motion of the wheel (see Roper, 2018), and so push the car to the *left*, i.e. forwards (Note 3). The ground doesn't know anything about the rest of the car; it just feels something pushing on it to the right and pushes back on it to the left.

Fig. 3: If the car is driving to the left, the wheel must be rotating anticlockwise, which means that the bottom of the tyre is pushing on the ground to the right. Friction from the ground resists this by pushing to the left, moving the car forwards.

I think that teachers often believe that all of this is too much complicating detail for students. Often books or teachers try to avoid the complication of wheels by putting a mysterious 'driving force' arrow in the air above the vehicle (Figure 4), instead of showing the actual forces acting on the wheels. This gets around the problem of having to think about the forces at all *four* wheels, which in general won't be equal. This is a good example of a simplification that is well intentioned but may actually lead to more problems than it solves, and may be partly responsible for students' confusion in this topic. We end up imagining the driving force on a toy car being provided by a child who pushes it along with their hand. It's a good example of making something harder by trying to make it easier. It isn't obvious why an arrow hovering over the car (like the child's hand) wouldn't work just as well for a car driving over ice. After all, there's less friction on ice to slow the car down, so shouldn't it go even *faster*?

Fig. 4: A mysterious 'driving force' coming 'from the engine' somehow propels the car forwards.

Students experience fewer problems from detailed explanations than they do from apparently 'simple' ones which turn out to fall apart when you think about them. Warren (1979) puts it so well that I will quote him in full:

> A teacher who has learnt an incorrect method may think that he understands it, and that it is easy, because frequent repetition has made it familiar to him and his own self-assurance convinces his students. He regards it as correct because it gives what is universally agreed to be the right answer. When he is told that the method is wrong and that he is confusing his pupils, he cannot comprehend the criticism. The effort of learning what is to him a new approach makes a sound treatment seem more difficult than the familiar unsound one. He does not see that, for a pupil who has not yet been taught any method, an approach … correctly applying Newton's laws, will in fact be just as easy to memorise and infinitely more easy to understand than an unsound method. It is astonishing to find how many people believe that pupils 'understand' analyses which are, in fact, incoherent nonsense, simply because they can reproduce them on demand. (p. 41)

The third law of motion

Now that we are zooming in on these pairs of equal but opposite frictional forces, we are implicitly using Newton's *third* law. Although students may claim that we never use this law either, we actually use it all the time. I think that the third law is generally the least well understood of the three. It is the third law that explains why the road decides to provide such a helpful external force to move the car forwards. It explains why a car needs an engine (in addition to a road), if all of the force moving it forwards actually comes from the road!

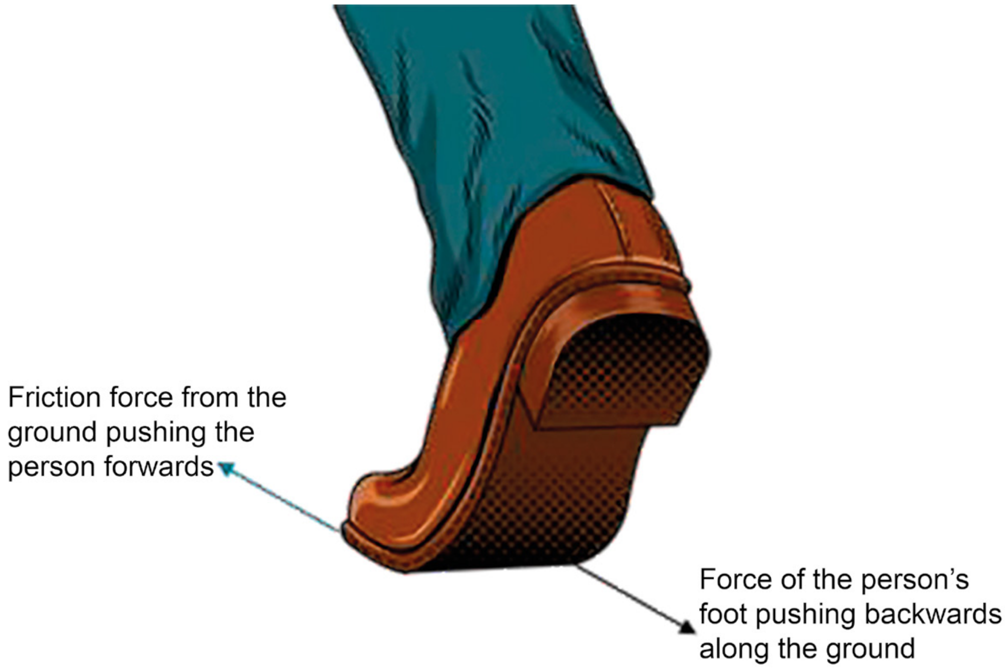

Fig. 6: Someone using friction to walk forwards

Newton's third law is often stated as: *To every action there is an equal and opposite reaction*. For me, the language of 'action' and 'reaction' is a bit old-fashioned and vague. It is also very readily misunderstood, because students wonder why these two equal and opposite forces don't cancel each other out. Why doesn't every tug-of-war competition necessarily end in a draw? The classic question about a car and a trailer is a good way to discover if students are confused about the third law:

A car is attached to a trailer (Figure 5). According to Newton's third law, if the car pulls to the left, the trailer pulls with an equal force to the right. So how can either of them ever move?

Fig. 5: A car towing a trailer

I prefer this version of the third law: *If an object A exerts a force on object B, then object B exerts an exactly equal but opposite force on object A* (see Roper, 2018). This version has the benefit of clarifying that the two forces always act on *different objects*, and therefore can never cancel each other out. If they *ever* cancelled each other out, then they would *always* cancel each other out, all objects would be perpetually in equilibrium, and all acceleration would be impossible!

The third law tells us that forces always come in pairs, a bit like the North and South poles of a magnet. Just as there are no magnetic monopoles, you will never observe a single, isolated force. Whenever you refer to a force, or draw an arrow to represent one, you should always ask yourself what/where its equal and opposite partner is.

Walking is similar to a rolling wheel. When you walk along the ground, you push your foot back against the ground, and the purpose of doing this is to benefit from the Newton's-third-law partner force, from the ground onto your foot, that pushes you forwards (Figure 6). It isn't just rockets and jet engines that rely on the third law to move – everything does!

The harder you push back on the ground, the harder the ground pushes back on you. Eventually, a limit is reached, which is the maximum force that these particular two surfaces can exert on each other – beyond that, your foot will slip.

Exactly the same thing is happening with the car. The whole point of the engine consuming fuel to drive the wheels around (anticlockwise in Figure 7) is to make those wheels push backwards along the ground (black arrows). That results in the ground pushing in the opposite direction on the wheels (red arrows), moving the car forwards. The stronger the push from the car on the ground, the stronger the pushback from the ground on the car – up to the friction limit of the ground/tyre contact (Note 4).

Fig. 7: The car wheels push on the ground (black arrows) and the ground responds by exerting a force on the car (red arrows) that pushes it forwards (similarly at all four wheels).

With the car and the trailer (Figure 8), if we put in the forces on the car (red arrows), road (black arrows), tow

forces. A person's weight is not a *contact force*, like all of the forces that we have considered till now. Weight is a *gravitational* force, which is 'action at a distance'; your weight is a force of attraction on you from the earth, and is exactly the same when you are in the air, mid-jump, as it is when you are standing at the same height up a ladder.

Your weight only ever acts on you; it can't act on any other object. Weight is the pull of the earth on the person, so the third-law partner force must be the pull of the person on the earth. As we saw with the black forces in Figure 8, everyday-size forces on the earth have no noticeable effect, because the earth is so massive. But if a lift is accelerating then the force of the person's feet on the lift floor will be different from their weight, and so we have to be careful to distinguish them. For example, if the lift had a downwards acceleration *greater* than the acceleration due to gravity, then the force of the person's feet on the lift floor would be zero, and the person would lose contact with the lift floor and be in freefall (until they hit their head on the roof of the lift!). But they would not truly be 'weightless', because they would still be being pulled down to the earth with the same force (their weight). They would be 'reaction-force-less', rather than 'weightless'.

It might be argued that we should leave these kinds of 'details' for the science department to sort out. But not every mathematics student studies physics. And I think that there isn't much value doing calculations with Newton's second law if students don't appreciate why those calculations are the right ones to do or what is actually going on. Teaching with conceptual understanding is always important in mathematics, and when teaching mechanics that must surely mean teaching about these kinds of ideas. Roper (2019b) argues for the benefits of practical demonstrations and experiments to support this.

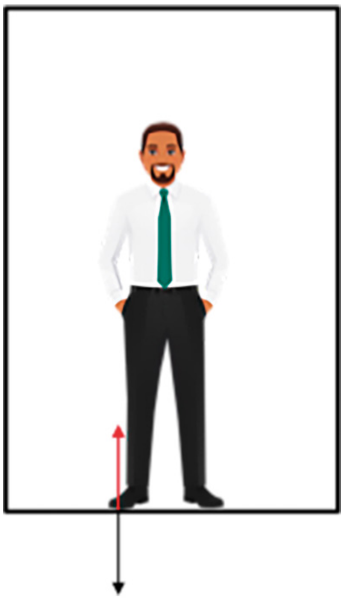

Fig. 10: A pair of equal and opposite contact forces at the bottom of a lift (elevator). Neither of these forces is the person's weight.

Notes

1. Roper (2018) points out that Newton's second law was originally expressed as force being equal to the rate of change of momentum, which has the advantage that it can also allow the modelling of variable-mass situations.
2. It is possible to have similar discussions about projectile motion. At the end of a set of questions on projectiles, you might include a question like this: *A bird takes off from a bird table at 3 m/s at 40 degrees to the horizontal. Where does it land?* Students will see that it is impossible to answer this question, but may struggle to say why, beyond the fact that a bird has a brain and free will.
3. We are ignoring here any deformation of the tyre as it rotates. What we are saying would be more accurate for a solid, wooden cartwheel than for a tyred wheel.
4. In a similar way, the winner in a high-jump competition can be thought of as whichever person can persuade the floor to exert the largest upwards force onto them. (Assuming that they are all of equal mass.)
5. Of course, I am making lots of simplifying assumptions, such as an absence of any resistive forces, such as air resistance.

References

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