Quotative and Partitive Models of Division

by **Colin Foster**

Like many people, I have always had difficulty with the distinction between *quotative* (grouping) and *partitive* (sharing) models of division (see Simmons, 2017, for an excellent explanation). The idea is that *sharing* 6 apples between 2 people, where everyone gets 3 each, is a different division process from *grouping* 6 apples so that everyone gets 3, and concluding that there must therefore be 2 people. In the first case, we work out how much of a *share* each person gets (3 apples each), and in the second we work out how many *groups* of apples there must be (2 people).

This has always felt to me like a distinction without a difference, and simply an artifact of perspective and choices of wording. Can't you always reconceive of any sharing as a grouping and any grouping as a sharing? Isn't it arbitrary which you consider each to be? You can tell the story either way round. If this is right, then it seems to me that quotative/partitive or grouping/sharing is just a lot of fuss about nothing – a pointless 'division of division' into two not-actually-different kinds, with confusing and hard-to-remember names thrown in, just to make it worse! Or am I wrong, and is this just the 'curse of knowledge' talking, where I am so familiar with division that I can't see the difficulties?

I was reminded of this issue while working at the dining table while my daughter and her friend Usha (both aged 5) were drawing.

My daughter said:

"I need some paper for Usha and me. I'm going to get four sheets."

I said, "Why four?"

She replied: "Two for now and two for later. No, I mean two for me and two for Usha. No, two for us both now, and then two for both of us later. You see?"

I saw. And this made me think about quotative and partitive division and how she had rather elegantly shown their equivalence. One interpretation of what she was saying was that she seemed to be alternating between these two interpretations of division. Since it happens to be the case for her example, $\frac{4}{2}=2$, that 2 is both the divisor and the quotient, I will modify the scenario a bit for clarity, and imagine that the 2 girls were each going to do 3 drawings each (at 1 pm, 2 pm and 3 pm), instead of 2. Now, for each calculation, $\frac{6}{2}=3$ and $\frac{6}{3}=2$, *in exactly the same real-world scenario*, we can tell either a grouping story or a sharing story, whichever we please:

Calculation	**Sharing story**	**Grouping story**
$\frac{6}{3}=2$	We share 6 sheets of paper among 3 occasions. How many sheets of paper are there on each occasion? $\frac{\text{6 sheets of paper}}{\text{3 occasions}}$ = 2 sheets of paper per occasion	We distribute 6 sheets of paper so that each person gets 3 sheets. How many people are there? $\frac{\text{6 sheets of paper}}{\text{3 sheets of paper per person}}$ = 2 people
$\frac{6}{2}=3$	We share 6 sheets of paper between 2 people. How many sheets of paper does each person get? $\frac{\text{6 sheets of paper}}{\text{2 people}}$ = 3 sheets of paper per person	We distribute 6 sheets of paper so that there are 2 sheets of paper on each occasion. How many occasions are there? $\frac{\text{6 sheets of paper}}{\text{2 sheets of paper per occasion}}$ = 3 occasions

I stress again that in each of these four stories there is *exactly the same scenario* of 2 people, each with 3 sheets of paper. I am *not* changing the number of people or the number of sheets of paper per person:

		Occasion		
		1 pm	2 pm	3 pm
Person	*Maya*	1 sheet of paper	1 sheet of paper	1 sheet of paper
	Usha	1 sheet of paper	1 sheet of paper	1 sheet of paper

OK, you may suspect that this is a carefully-contrived, awkward example. But, as far as I can see, every example is basically like this, and any difference is just superficial, to do with the names of the particular things being shared or grouped and how you choose to look at it. If the units are people, 'sharing' stories may seem more natural (sharing among 'occasions' may feel a bit artificial), but that is just an artifact of how we think about it. There is no fundamental difference between grouping and sharing – it is all in the eye of the beholder. It seems completely symmetrical to me, so, in the same context, it's arbitrary whether you call either of the divisions 'sharing' or 'grouping'. Maybe the reason that people find this distinction difficult is that it is fundamentally spurious?

If my daughter were to distribute the sheets of paper one by one: "one for you, one for me – that's our first pair of drawings – then another one for you, and another one for me – that's our second pair of drawings", then this is either *sharing* out among the people, or *grouping* into pairs of drawings, and these are identical processes. I don't see how you can say that it's more one of these than the other. Sharing things among n people is precisely equivalent to putting them into groups of n and seeing how many groups you get, because the number of groups is exactly the same as the size of each share.

I wonder how much classroom time has been spent trying to persuade children of this odd distinction, or trying to convince trainee teachers that they don't really understand division as well as they think they do, because they fail to use the 'right' words in the right places when talking about grouping and sharing? Asking someone whether a particular scenario is grouping or sharing is an impossible question, because any scenario can be viewed as either. I think this is something we should stop worrying about.

Reference

Simmons, M. (2017). The hidden faces of division. *Mathematics Teaching, 259*, 16-19.

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