

gratitude and honour of Malcolm. We will leave the last word to him:

“The purposes we have in mathematics teaching are broad; including procedural fluency, conceptual understanding, strategic competence in both pure and applied problem solving, and a critical awareness of the quality of mathematical reasoning. Each purpose requires a range of appropriately designed mathematical tasks” (Swan, 2014). Hard to argue with that!

## Further Reading

Evans, S. and Swan, M. 2014 ‘Developing Students’ Strategies for Problem Solving in Mathematics: The Role of Pre-designed “Sample Student Work”’, *Educational Designer*, 2, 7. <http://www.educationaldesigner.org/ed/volume2/issue7/article25/index.htm>

Swan, M. 2015 ‘Designing Formative Assessment Lessons for Concept Development and Problem Solving’. In Bartell, T. G., Bieda, K. N., Putnam, R. T., Bradfield, K., and Dominguez, H. (Eds), *Proceedings of the 37th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 5–8 November 2015, East Lansing, Michigan, USA.

Swan, M. and Burkhardt, H. 2014 ‘Lesson Design for Formative Assessment’, *Educational Designer*, 2, 7. <http://www.educationaldesigner.org/ed/volume2/issue7/article24>

## References

Clements, D. H. 2002 ‘Linking Research and Curriculum Development’. In English, L. D. (Ed.) *Handbook of International Research in Mathematics Education*, pp. 599–636, Lawrence Erlbaum Associates, Mahwah, NJ.

Swan, M. 2014 ‘Designing Tasks and Lessons that Develop Conceptual Understanding, Strategic Competence and Critical Awareness. *Tarefas matemáticas: Livro de Atas do Encontro de Investigação em Educação Matemática*, pp. 9–28.

## Notes

1 See <http://www.cambridgemaths.org/news/view/what-is-a-curriculum/>

2 See <http://www.mathshell.org/swan.htm>

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**Author** Lucy Rycroft-Smith.  
**e-mail:** [lucy.rycroft-smith@cambridgemaths.org](mailto:lucy.rycroft-smith@cambridgemaths.org)

*Lucy Rycroft-Smith works in Communications and Research for Cambridge Mathematics ([www.cambridgemaths.org](http://www.cambridgemaths.org)).*

# REVIEWS • REVIEWS • REVIEWS • REVIEWS • REVIEWS

### **The Call of the Primes: Surprising Patterns, Peculiar Puzzles, and Other Marvels of Mathematics**

Owen O’Shea

Prometheus Books, 59 John Glenn Drive, Amherst, New York 14228, USA

<https://www.prometheusbooks.com>

ISBN: 978-1-63388-148-8

330 pages

\$19.00

Owen O’Shea’s book aims to excite and intrigue readers into the beautiful world of mathematics. Its blurb includes this statement: “This is the perfect book for people who were turned off by math in school but now as adults wonder what they may have missed.” Even if the readers this book aims to cater for were turned off by mathematics, the writer seems to assume that they have a reasonable aptitude towards mathematics – it doesn’t take long before the reader comes across some mathematics that will challenge them; however, any mathematical understanding beyond GCSE level is introduced relatively gently.

The book takes the reader on a tour of the most commonly covered recreational mathematical topics: magic squares, primes, Pythagorean triples, the Monty Hall problem, the Fibonacci sequence, Lucas numbers, square and triangular numbers, Pascal’s triangle, an introduction to combinatorics and permutations, Phi, imaginary numbers,

$\sqrt{2}$ ,  $\pi$ , and  $e$ . The reader is introduced to proof in a very non-threatening manner; many results throughout the book are proved, although there are several ‘facts’ that are thrown in with no proof or reference link to other reading.

The only disappointing aspect of the book is the repeated mentions of numerology and his frequent emails with Dr. Cong, the “well-known Chinese numerologist and number expert”. These coincidences, whilst interesting, are never called out to be coincidences. For example in the chapter on square numbers: “Bobby Fischer won the US Championship in chess in the 1957/58 season when he was just 14 years old. He won the World Championship in chess in 1972 when he was just 29 years old. Consider those two numbers, 14 and 29. 14 multiplied by 29 equals 406. The 406th prime number is 2791. Reverse the digits of that number and one obtains 1972, the year Fischer won the World Championship.”

In short though this book is a pleasing read, the author’s love of mathematics comes across clearly despite it not being his specialism. As he writes in his introduction: “If mathematics is taught with emphasis given to its beauty, elegance, and order, then students and the general public will more often than not be tempted to learn more and appreciate it for the fascinating subject that it is.”

**Mark Crossley**

### **Mathematics Rebooted: A Fresh Approach to Understanding**

Lara Alcock

OUP Oxford

ISBN: 978-0-19880-379-9

£20, hardback

We all know that it’s an occupational hazard of being a mathematics teacher to encounter people in daily life who, on finding out what you do, say “Oh, I was never any good at maths at school”. What can you say in such a situation? On asking for more information, I am often struck by someone who appears to be perfectly competent at lots of other things, so that it is unclear why mathematics should be the exception. Well, I now have a possible answer, which is to suggest that they read Lara Alcock’s book, *Mathematics Rebooted*.

The potential readership of this book is actually much wider than that: people who liked mathematics up to a point, but then got lost; people who gave up mathematics to pursue something else; people who studied mathematics beyond school, but feel that they have forgotten it all; teachers whose specialism is not mathematics; even keen school pupils, and keen parents! And I would add to this mathematics teachers who want to think through some interesting ways of approaching parts of the school mathematics curriculum.

If you are familiar with Lara's other books (How to Study for a Mathematics Degree, How to Think About Analysis), then you will immediately recognize her precise but friendly writing style. Every word is chosen carefully, and the diagrams (of which there are many) are clearly and attractively presented, and interposed elegantly throughout the text (none of that cumbersome "see Figure 1" nonsense). To read this book is to be accompanied through some beautiful mathematics by an enthusiastic expert teacher, mathematician and mathematics education researcher, whose delight for the subject is evident on every page.

The material is ambitious and the journey is compelling, but at no stage does it feel forced. One thing naturally leads to the next, and it is only when you pause for breath that you see how far you have come. In Chapter 1, we are led swiftly from basic ideas of multiplication, using the array model to make plain the multiplicative structure of the integers, through areas, fractions and Pythagoras' theorem, culminating in a discussion of Fermat's Last Theorem. But the strength of this book lies not only in the mathematical content. Interwoven through each chapter are observations about such things as definitions, proof (including by induction and contradiction) and the power of algebra for making a general argument. And everything is supported throughout by insights from research in mathematics education and psychology.

Chapter 2 begins with tessellations, concluding by finding all the semi-regular tessellations, and along the way introduces symmetries and groups, as well as aperiodic Penrose tilings. Chapter 3 takes infinite series as a vehicle for examining addition of fractions, supported by extensive use of number lines, and finishing with the divergence of the harmonic series. Chapter 4 begins with line graphs and linear programming and proceeds to curves and 3D, and Chapter 5 looks at integers, divisibility and decimals, and finishes with Cantor's diagonal argument for the uncountability of the reals.

This book is a delightful journey through a lot of stimulating mathematics. I enjoyed the many asides, some of which are gems, such as a digression on calculators, which Lara describes as "fast but stupid" (p. 106). Throughout the book, the pleasure of doing mathematics comes through powerfully, such as when Lara writes about "the satisfaction that comes from understanding why an argument works" (p. 80). If this book cannot help to bring a lost soul back to mathematics, then nothing will. In its style it reminded me of reading W. W. Sawyer's books, such as *Mathematician's Delight*, when I was a teenager. This is an excellent book that could give a lot of pleasure and insight to an interested student, as well as

to those who, for one reason or another, have lost touch with mathematics over the years. Besides that, there is much here for mathematics teachers to ponder, in terms of how topics might be introduced and built up in a coherent way.

Colin Foster

**Cuisenaire – from Early Years to Adult**

Mike Ollerton, Simon Gregg,  
Helen Williams  
Association of Teachers of  
Mathematics  
Vernon House, 2A Vernon Street,  
Derby DE1 1FR  
[www.atm.org.uk](http://www.atm.org.uk)  
ISBN: 978-1-898611-97-4  
Book and Download (74 pages)  
Non-members £16, Members £12

Invented in 1945 by the Belgian teacher Georges Cuisenaire the rods of different lengths and colours, named after him, were designed to develop understanding of numerical relationships. Their use was championed by Caleb Gattegno (a founder member of the ATM).

*Cuisenaire – from Early Years to Adult* came about following a discussion at an ATM conference and the subsequent development of ideas. It has led to a beautifully illustrated book showing ideas to engage with, illustrations of how to use the rods and photographs of pupils engaged in the tasks. The topics covered include counting, sequences, patterns and algebraic reasoning, fractions, ratio and proportion, arithmetic, and problems to "see the interconnectedness of mathematics".

Ollerton, Gregg and Williams encourage the idea of play and then ask numerous questions ... "what can you make?", "what have you found out?", "what do you know?", ... to encourage analytical thought. There are ideas to engage young learners in group work and individual study.

'Staircases' and 'trains', in which rods are stacked or concatenated, leads to ideas of number patterns and the reader learns how pupils approached these problems and the articulations of their findings. The final 'train' task produced a Fibonacci-looking pattern which nicely led onto the next section developing ideas of sequences and algebraic reasoning. The fourth section is on fractions, ratio and proportion and the next ones are on addition and subtraction, and multiplication and division. PPPs (product and partner pairs, for example,  $8 \times 1$ ,  $4 \times 2$ ,  $2 \times 4$ ,  $1 \times 8$ ) leads to the plotting of points on a graph and encourages explorations of negative numbers, square roots and

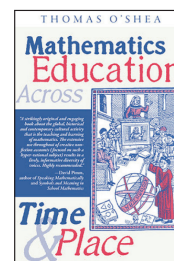
rectangles with constant perimeters. The final sections have ideas to develop more complex mathematical concepts including 'hollow triangles' which lead onto Pythagoras and trigonometry.

*Cuisenaire – from Early Years to Adult* is yet another excellent resource from ATM. This broad collection of tasks is full of interesting ideas and open-ended questions to encourage learners to explore mathematical concepts and it is supported by colourful pictures and photographs and pupils' responses. There is also a set of very useful whiteboard slides to go with the book. Ollerton, Gregg and Williams say "use the ideas to deepen the understanding of your learners" – this is an aspiration which their book should help to achieve.

John Sykes

**Mathematics Education Across Time & Place**

Thomas O'Shea  
Friesen Press, Suite 300-900 Fort  
Street, Victoria, BC, Canada  
[www.friesenpress.com](http://www.friesenpress.com)  
978-1-4602-8608-1 (hardcover)  
978-1-4602-8609-1 (paperback)  
978-1-4602-8610-4 (eBook)  
\$34.99, \$20.99 & \$9.99 (respectively)



The book covers mathematics education from ancient Greece through to the 20th century. The question of what should be taught is an age-old question also asked by teachers in ancient Greece, 3000 years

ago, in such places as Plato's Academy or Aristotle's Lyceum. This is followed by the Roman era which introduced the seven liberal arts and sciences of the trivium and quadrivium, the latter as advanced studies. It is fully referenced for further study.

Throughout the book, each era is enhanced with what appears to be primary documents to back up the evidence. Although purely fictional, they also get the main points of the chapter across to the reader in a meaningful manner. The 'story' continues through the Islamic period, the Renaissance and then Canada (where the author resides) concluding with the 20th century in New York, England and Tanzania.

The English mathematicians are probably less well known than the usual characters that appear in many historical books of mathematics such as Hardy, etc. They include Cabber, Bourne and Mercer, all from the 16th century. They are