Teaching the **BIG Ideas** in School Mathematics

By **Colin Foster**

It isn't unusual to hear mathematics teachers despairing at the amount of content they have to get through in the limited time they have with their students. Much of this material can be perceived as a list of 'things they have to be able to do', rather than a deep engagement with important and substantive ideas. Many mathematics lessons seem to begin with the teacher effectively saying, "OK. Well here's another thing you need to be able to do." And the lesson consists of the teacher demonstrating this particular technique, followed by the students rehearsing it until they can perform it efficiently and successfully (Foster, 2025). If this is achieved before the bell goes, then it was a 'good lesson'.

The pressure of this enormous list of content means there may be little space for students to ask questions or think more deeply about the mathematics, not because the teacher doesn't care about this, but because there are just so many more techniques to get through in the very limited amount of teaching time available. The teacher can't afford not to 'cover' everything, as there are always high-stakes examinations on the horizon (or nearer), and no one wants their students to be disadvantaged by inadequate preparation – so what else can be done?

I think there are three main problems with this:

- *First*, getting through all the curriculum content this way consumes a lot of time. Every individual process that the students need to be able to do has to be separately taught, and repeatedly reviewed, so they reach a solid level of performance, and don't subsequently forget how to do it. Lots of time has to be devoted to retrieval practice of previously learned content, otherwise it will be lost and have to be taught all over again. No one has much time left over for things like problem solving.
- *Second*, the students' motivation for getting on board with this approach relies on the satisfaction they will receive from mastering these individual skills, and eventual success in a high-stakes exam, which may be some years away. This works for some students, but not all, meaning that schools end up feeling forced to resort to imposing unpleasant consequences on students who fail to get with the approach. Mathematics becomes seen by everyone as a chore that must be done, largely because the consequences of *not* doing it are worse.
- *Third*, and for me most importantly, this approach to learning mathematics fails to convey to students much of the real nature of doing mathematics. 'Conscientious' students, who perhaps even enjoy learning lots of procedures, may well be successful in their exams. But even if they are, they may not leave with a positive and realistic view of what using mathematics in further study or in a career is going to be like. Learning mathematics in a procedure-focused way isn't at all typical of what doing mathematics is likely to look like beyond the artificial school environment. And students who don't like the way school mathematics is done may turn away from the subject without ever having imagined what a more authentic mathematical experience could look like.

Some might say that the solution requires a slimming down of the school mathematics curriculum, but I think there are reasons to resist this. Those progressing to further study really need many of the things in it, and we'd want all students to experience many of the great highlights of mathematical thought, even if they end up not formally pursuing the subject further. So, I prefer to think about whether it could be possible to re-package what is in the existing curriculum differently. Rather than attempt to separately train each of these micro-skills to confident performance, endlessly practising each one, so as not to forget how to do them, I have been trying to think about how a more 'big picture' approach might work in practice. It is easy to say "Don't be procedural", but when there are lots of procedures students need to learn, what else can you do?

Seeing the bigger picture

It has been estimated that there are around 200 'procedures' that school mathematics students typically need to master to be successful in compulsory examinations at age 16 (Note 1). As a perhaps terrifying thought experiment, I have been trying to imagine what teachers would do if, instead of 200 of these procedures, there were, say, 2000 or even 20,000 such procedures that students were going to be expected to be able to perform, on demand, under examination conditions, by that age. As the number of target procedures goes up, at some point we would have to accept that there could never be time to teach each of those procedures separately, one after another. We would have to think of something else to do.

What would teachers do? I think what would happen is that teachers would carefully analyse those 2000 or 20,000 procedures and look for their commonalities. What do all those separate procedures share with one another? Which procedures are basically minor variations on other ones? What much smaller number of big-picture ideas could underlie success with all these procedures? What is, in theory, the *least* amount we might have to teach in order to set up a student for success with this huge number of desired procedures, many of which they would never encounter in that exact form on the day of the examination, or ever?

Thankfully of course, we don't have 20,000 procedures to teach, but 'only' around 200. And perhaps it *is* do-able to teach each of those 200 procedures one by one. This is after all what I think most schools currently attempt to do. However, to do it, teachers have to hurry, and little time is left for bringing the ideas together and exploring the connections. Students and teachers alike often describe learning mathematics as a pressured, stressful experience, constantly against the clock. But what would happen if we treated our 200 procedures as though they were a functionally infinite number? What would school mathematics look like if we tried to identify those small number of key, bigger ideas, and focused most of the teaching time on those instead? Could that enable more sense-making and render it unnecessary to spend so much time on each of the separate procedures that derive from them?

Focusing on five BIG Ideas

I have been thinking for some time whether this could work for teaching mathematics, and if so what the *BIG Ideas* would be. I am sure there is not one right answer to this, and perhaps it is more important to have *some* set of fundamental ideas to focus on, rather than exactly what they are. But, to be concrete, I have eventually settled on the five *BIG Ideas* shown in Figure 1: *Thinking Multiplicatively, Thinking Algebraically, Thinking Geometrically, Understanding Functions and Graphs,* and *Modelling.* For me, these five *BIG Ideas* are the structure that support the entirety of school mathematics.

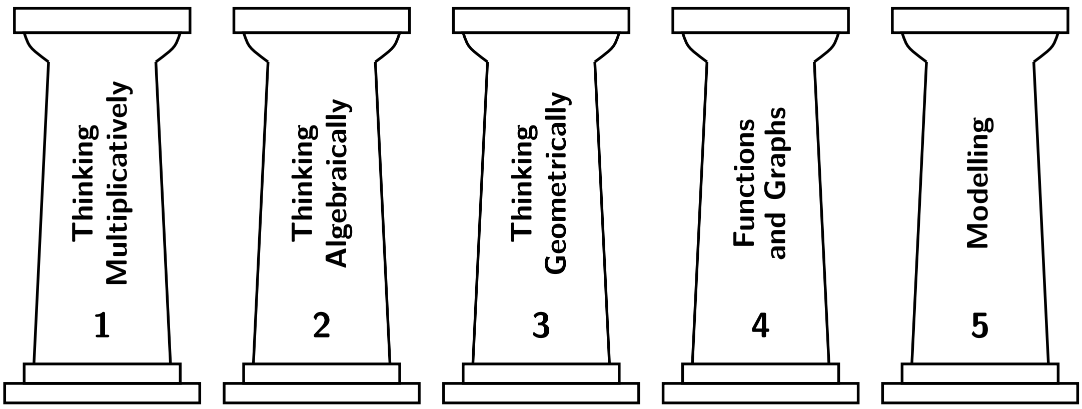

Figure 1. The five BIG Ideas of school mathematics

Of course, there's nothing earth-shatteringly novel about these *BIG Ideas,* and every mathematics teacher is already well aware of them and teaching them! All I am suggesting is that we give them very special priority over everything else, and spend a disproportionate amount of time on them. Everything has importance, but you can't focus on everything. By teaching these *BIG Ideas* slowly, carefully and really deeply, students will obtain the keys to a surprisingly large amount of the entire curriculum. Once a student has a profound grasp of these five *BIG Ideas*, it is quite suprising to appreciate how many areas of mathematics become essentially minor variations on a familiar theme, rather than something new.

I am not saying we *only* teach the *BIG Ideas*, and leave students to work out all the details for themselves. Certainly not! We teach everything. But I'm advocating that we *primarily* teach the *BIG Ideas*, again and again, really focusing on getting those ideas deeply understood, so all students know them inside out. And then we can, much more briefly, show students how everything else they need follows, hopefully without too much difficulty, from them. How many times have students struggled through a topic such as trigonometry, learning very little that will stick, largely because they are overwhelmed with difficulties they have with things like ratio and proportion, functions and graphs, and so on?

Thinking multiplicatively

I will focus here on just the first of the five *BIG Ideas*, *Thinking Multiplicatively*, to try to illustrate how much you can get from this one idea. There is lots I could say about *Thinking Multiplicatively*, but for me its essence is that we can get from any non-zero number a to any other number b by multiplying by $\frac{b}{a}$ (Figure 2).

$$a \xrightarrow{\times \frac{b}{a}} b$$

Figure 2. We can get from any non-zero number a to any other number b by multiplying by $\frac{b}{a}$.

That sounds very abstract, and I am by no means saying we just tell this to students and our work is done! Of course, they have to see this in many guises, many examples, many contexts, and develop fluency finding a or b or $\frac{b}{a}$, given the other two. But a lot of school mathematics can be seen as an application of this. When the preparatory work of learning to *Think Multiplicatively* is complete, students respond to many new topics by saying "Oh, it's just multipliers again!" A student once said to me, "Is that all maths is – just multipliers?" Of course, the answer is 'no', but I was pleased with the question, because it suggested they were seeing these connections across many topics.

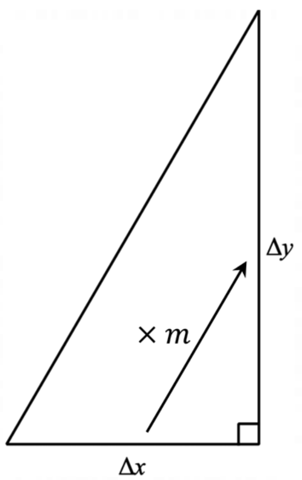

Figure 3. Gradient is just the multiplier that takes the x-increase to the y-increase.

For example, imagine if, when students met the idea of gradient, they were already very familiar with *Thinking Multiplicatively*. Then, gradient is just a *particular* multiplier – the one that gets us from an x-increase to a y-increase (Figure 3). Everything they already know about multipliers makes this a natural idea to understand.

Students will often describe gradients using 'angle language', such as by saying that a $y = 3x$ line is at a 'steeper angle' than a $y = x$ line. Is it 'three times' steeper? They might measure the angles between the line $y = mx$ and the x-axis for different values of m, and be shown that the inverse tan button on the calculator (provided it's in degrees mode) gives the angle. The tangent of that angle is nothing more or less than the gradient m, the multiplier that takes the base to the height (Figure 4). The gradient and the tangent of theta are different names and symbols for the same thing.

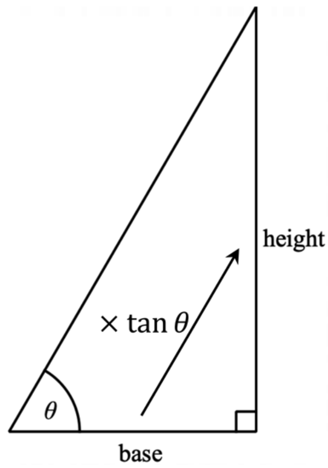

Figure 4. The tangent of the angle θ is just the multiplier that takes the base to the height.

All the trigonometric functions can be thought of as multipliers like this. So are lots of other things (Figure 5).

$$\text{time} \xrightarrow{\times \text{ speed}} \text{distance}$$

$$\text{diameter} \xrightarrow{\times \pi} \text{circumference}$$

$$P(A) \xrightarrow{\times P(B|A)} P(A \cap B)$$

Figure 5. Just a few of numerous possible examples of multipliers from the school mathematics curriculum.

The more you think about it, the more examples you discover. A lot of things that are taught as separate topics can be thought of as another application of multipliers. This can make teaching these topics much more efficient and successful. The 'hard bits' of so many topics turn out to be the same difficulties with *Thinking Multiplicatively*. Spend enough time on this, and you are feeding many birds with one scone.

When you can *Think Multiplicatively* with confidence, there isn't much left to many topics, including everything to do with fractions, percentage change, ratio, gradient, trigonometry, statistical charts and most formulae from school science. By focusing on the concept of the multiplier as a stretching/scaling factor, rather than just repeated addition, we find that previously disparate topics start to look 'the same'.

For example, dividing fractions is no longer a strange rule to be memorised, but a simple case of realising that in Figure 6, dividing by $\frac{b}{a}$ (going from right to left, instead of left to right, as in Figure 2) must be equivalent to multiplying by $\frac{a}{b}$, because we are now going from b to a, instead of from a to b.

$$a \xleftarrow{\div \frac{b}{a}} b$$

$$a \xleftarrow{\times \frac{a}{b}} b$$

Figure 6. Realising that $\div \frac{b}{a}$ must be equivalent to $\times \frac{a}{b}$.

Conclusion

What I am presenting here is not really a convincing argument unless you think through the details and work it through for every topic, and I have tried to do this on a completely free website: **bigmathematicalideas.org**. There, I explore in detail what exactly each of the five *BIG Ideas* consists of, and then I have a big section "What does Thinking Multiplicatively get us?", and similarly for every other *BIG Idea*, unpacking the different topics that fall out as a result of that *BIG Idea*. The topics will seem 'all jumbled up' across the different *BIG Ideas*, rather than organised as you might expect to find them. Actually, I would argue they are not jumbled up at all, but sorted out 'properly', exactly where they belong! I also try to show with lots of tasks and examples how a deep and connected understanding of each *BIG Idea* facilitates problem solving across a variety of content areas.

I wouldn't want to stereotype mathematics teachers, but I think a lot of us are detail-oriented kinds of people. That can mean we are very good at attending to the smaller details of individual procedures and lesson objectives, but sometimes struggle to step back and think at a bigger-picture level. Having said that, a *BIG-Ideas* approach like this will not be novel for all teachers. It is what any teacher would do if they had an unusually limited timeframe to teach someone, such as with a student who has missed a large amount of school due to personal circumstances or illness, or who is resitting GCSE mathematics in a year to try to get a better grade. In these situations, the teacher has to prioritise and condense the course down to its essential points, because they simply don't have the luxury of going through every separate detail. I think I'm saying that the kind of thinking involved in doing that, and the practice of distributing teaching time in that kind of way, is likely to be beneficial for *all* students.

Note

1. This is obviously very much a ballpark figure, as who can say really what should count as an individual, separate 'procedure'?

Reference

Foster, C. (2025). 'Alternatives to atomisation', *Mathematics Teaching*, 295, pp. 37–41. www.foster77.co.uk/Foster,%20Mathematics%20Teaching,%20Alternatives%20to%20atomisation.pdf

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Geometry Problem 32

By Catriona Agg

All four triangles are equilateral.
What fraction of the largest triangle is shaded?

Send your solutions to chrispritchard2@aol.com, and we will publish the best.