The amber traffic light

By **Colin Foster**

Many real-life situations can be used as a basis for doing some mathematics, but very often these pseudo-real-life contexts are not very plausible. In many cases, the mathematics isn't really needed, and feels bolted on for the sake of doing mathematics. In real life, often it's enough to use common sense and informal judgments, and calculations aren't actually needed. So, I'm always interested in finding examples where the answer isn't obvious without some calculation, and mathematics really helps. In these kinds of mathematical modelling situations, doing the calculations is always the easy bit: the challenge is deciding what information you need and how to manipulate it.

Someone was recently talking about driving through traffic lights. In the UK, traffic lights show a single amber light as a warning that a red (stop) light is about to come on. According to the *Highway Code* "AMBER means 'Stop' at the stop line. You may go on only if the AMBER appears after you have crossed the stop line or are so close to it that to pull up might cause an accident" (**www.highwaycodeuk.co.uk/light-signals-controlling-traffic.html**). Inexperienced drivers will sometimes jam on the brakes hard as soon as they see an amber light, but this can be more dangerous to following traffic than just continuing through the junction, if they are sure that they can make it through safely before the red light comes on.

This decision requires quite a bit of judgment. Should you slow down when approaching a green traffic light, in case it turns to red before you arrive? Or can you proceed as normal until the moment that you see an amber light, and only slow down then if necessary? Presumably the answer depends on the speed at which you are travelling. You might like to pause here to think about what mathematics might help to answer these questions and what information you might require to do so?

Warning time

In the UK, the amber light stays on for about 3 seconds. People sometimes assume that it is set to stay on for longer on faster roads, but as far as I can discover this isn't the case – everywhere seems to be 3 seconds. So, one approach would be to say that if you are less than 3 seconds away from the traffic light when it turns amber then, if it's safe to do so, you should continue, and you will pass through the light before it turns red. This may be safer than braking harshly to stop at the lights. But if you're more than 3 seconds away from the light when it turns amber, then you *won't* be able to make it through without speeding up, and so you should stop. To follow this rule, there's no need to do any further calculation because you can practise counting the three seconds in your head until you can reliably estimate whether you can make it through or not.

But there's a second factor to consider, which is whether by following this rule you will always have enough space to stop safely before the light when you are more than 3 seconds away. Clearly this will depend on how fast you are travelling. You might be fine on a 30 mph road, but what about when you are travelling on a 60 mph dual carriageway that has traffic lights? On such roads, should you slow down from 60 mph when you see traffic lights, even if they are on green, so that you're prepared, just in case they might change to red before you get there? If so, then how slowly should you go? Or will you be OK continuing at 60 mph, because 3 seconds at 60 mph gives you twice as much space to stop in as does 3 seconds at 30 mph?

Point of no return

The other way to think about the problem is to imagine the distance in front of your car in which you can safely stop, at whatever speed you are travelling. As long as you are further than this stopping distance from the lights, you should be able to stop safely if the lights turn amber. But once you get closer to the traffic lights than this safe stopping distance, you have reached a 'point of no return', and so you should continue on, even if the lights change to amber.

This sounds like a risky strategy, because how can you guarantee that the lights won't have turned red by the time that you reach them? If there is a risk of this happening, then that must mean that you were travelling too fast, regardless of what the speed limit was. So, how fast is it safe to travel through traffic lights, so that the point of no return is guaranteed never to be more than 3 seconds away from the lights?

Solution

If you're travelling at, say, 50 mph, and are 3 seconds out from a traffic light, that means that the light must be 66.7 m away (see Table 1). Not many people seem to know that to convert miles per hour into metres per second you need to multiply by $\frac{4}{9}$, and so to do this students will probably have to convert miles → km → m and hours → minutes → seconds separately, which requires some careful thinking.

The *Highway Code* stopping distance for 50 mph is 53 m (Table 1), which is less than 67 m, so potentially you can stop before you get to the lights. But that assumes that you react immediately, and that the car and road conditions are good – and it doesn't allow for the possibility that someone is following too closely behind you, and won't be able to stop safely even if you do. Just because you *can* stop doesn't necessarily mean that you *should*.

As your speed increases, things get worse. You can see in Table 1 that by the time we get to a speed of 70 mph, the *Highway Code* stopping distance (96 m) exceeds the 3 seconds of space available (93 m) once the amber light comes on. So, whizzing through green traffic lights at 70 mph would seem to be much too fast, unless perhaps you have just observed them turn green and therefore can be confident that the amber light isn't about to come on.

It is often argued that tyres and brakes have improved over the years since the Highway Code values were published, and so these stopping distances are excessively cautious (**www.youtube.com/watch?v=xs04E3MabNU**). On the other hand, it's also argued that reaction times were underestimated in the Highway Code, and also may have got worse over time, given all of the electronic distractions present in modern cars. The road safety charity Brake (**https://www.brake.org.uk/**) has published updated stopping distances, based on research by the Transport Research Laboratory, which are considerably longer than those in the Highway Code (see final column in Table 1, **www.fleetnews.co.uk/news/company-car-tax-and-legislation/2017/07/25/brake-argues-highway-code-stopping-distances-should-be-extended**). Using the Brake values, 40 mph would look like the maximum safe speed for approaching green traffic lights that could turn amber at any moment. Beyond 40 mph, the Brake stopping distance exceeds the 3-second space available when amber comes on. And it bears repeating that these values are only applicable for good vehicles and road surfaces, in dry weather with an alert driver; otherwise, even slower speeds would be necessary.

Speed (mph)	Speed (m/s)	Distance covered in 3 seconds (metres)	*Highway Code* stopping distance (metres)	*Brake* stopping distance (metres)
20	8.9	26.7	12	19
30	13.3	40.0	23	34
40	17.8	53.3	36	51
50	22.2	66.7	53	71
60	26.7	80.0	73	95
70	31.1	93.3	96	121

Table 1. Speeds and distances

Conclusion

You might think that there 'wasn't much mathematics' in this article, but I think that depends on what you count as 'doing mathematics'. Clearly, there's a lot more to solving a problem like this than knowing 'speed equals distance divided by time' and converting units. It's obviously much more challenging to answer a question like this than a typical textbook question. But if we want students to be able to deal with problems that arise in life or work, which people actually care about the answers to, then they need opportunities to try problems like this, where the technical, mathematical requirements may be minimal but the modelling/thinking requirements are high.

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