Questions Pupils Ask!

What are 'like terms'?

by **Colin Foster**

I have watched a few lessons recently on 'collecting like terms', where pupils were simplifying expressions like $3a - 2b - a + 5b$ and $5a + a^2 - 4a - 3a^2$. These lessons left me questioning whether this is an easy topic or a hard topic? Or, perhaps it is one of those 'easy except for negative numbers' topics, where pupils can be very successful answering questions, provided that negative numbers don't make an appearance. Or, maybe it is one of those 'easy to do but hard to understand properly' topics.

Many pupils find simplifying expressions like $5a + 6b + 2a + 4b$ quite easy, perhaps invoking 'apples and bananas' thinking (for problems with 'letter as object', see Foster, 2019). Negative terms are sometimes OK, provided that, when working left to right, the running total never drops below zero. So, $5a + 6b - 2a - 4b$ would be fine, but $5a - 4b - 2a + 6b$ might not be.

Generally, the teacher exposition that I saw focused on breaking up the expression into separate, signed terms and reordering:

$$\boxed{+5a}\boxed{+6b}\boxed{-2a}\boxed{-4b} = 3a + 2b$$

But this is all about 'doing'. Is 'collecting like terms' perhaps one of those topics where 'doing' can easily outstrip 'understanding'? In one of the lessons I watched, I frequently heard statements like "You can only add things if they're *the same*" or "You can't add an a to a b". This seemed dissonant with the fact that pupils' answers were things like $2a + 3b$, which is apparently 'adding an a to a b'. Later on, this led to some queries, and the teacher tried to clarify this by referring to 'like terms': "You can only add *like terms*" (which might perhaps have been confusing, since pupils were also *subtracting* them). But, saying this seems to be the same problem as saying "You can't add a and b". You *can* add these things; the point is that you just don't end up with an expression that can be simplified.

Later on, a pupil asked, "What are 'like terms'?". The teacher said, "They're things that you can add together, like $2a$ and $3a$; they're the same *kind* of thing", which seemed rather circular. We have to look for 'like terms', so we can add them together, so what are 'like terms'? – they are things you can add together! This all came to a head later on during the whole-class discussion, when the teacher had an expression like $5a + a^2$ on the board. The pupils had answers like $5a^3$, $7a$, $8a$ and $10a$, and the teacher was asking, "Are a and a^2 *like* terms?" I wasn't sure how the students were supposed to know. They mostly seemed to think yes, and, when asked why, one pupil said, "Because they've both got an a in them". The teacher asked, "But are they the *same kind* of a?", which the pupil seemed mystified by. Finally, the teacher asked another pupil, "Is a squared *like* a?", stressing the word 'like', to which the pupil replied, "A bit" [See Note 1].

It seemed to me a very reasonable response, both in terms of the English, but also in terms of the mathematics. There are things about $5a$ and a^2 that are the same, and things about them that are different. The notion of what can be simplified is also rather tricky here. Is it right to say that $5a + a^2$ "can't be simplified"? It could certainly be written as $a(5 + a)$, and this is precisely because the pupil is right that a and a^2 are 'a bit' alike; they have something in common – a common factor of a. So, it doesn't even seem right to say that 'only like terms, when added or subtracted, give an expression that can be simplified' [See Note 2]. Indeed, whether something can be simplified or not depends on what else we might know: you might not be able to simplify $a + 2b$, but if you knew that b was equal to $3a$, say, then you could.

So, what *are* 'like' terms, and is this a helpful concept for pupils doing this kind of work? For me, I see like terms as *constant multiples of one another*, but that is a pretty hard idea, related, I suppose, to the more advanced notion of 'linearly dependent' terms, and especially so, as all the 'constants' are different from one another – there doesn't seem to be much that is 'constant' about them. They are constants if we view the a's and b's, etc., as *variables*, rather than specific *unknowns*, and treat all of these expressions as if they are functions of many variables. Then, the coefficients are just numbers, and so stay the same as the letters change their values. This seems like a hard but important idea, which I didn't see any trace of in any of the lessons that I watched. What we are doing is finding simpler-looking, more convenient expressions for these quantities that will give the same values when you substitute in *any* choice you like of values for a, b, etc. So maybe we should be stressing this aspect, getting students putting various different numbers into these expressions to check their answers/conjectures, and maybe even writing these as identities: $5a + 6b - 2a - 4b \equiv 3a + 2b$?

Notes

1. Perhaps it seems strange to students that $3a$ and $-17{,}625a$ are 'alike' but $3a$ and a^3 are not 'alike'?

2. It can also be difficult for pupils to accept that 'simpler' can mean 'more complicated' from their point of view. For example, $3a$ may indeed be 'simpler' than $5a - 2a$, but $a(5 + a)$ may well seem more complicated than $5a + a^2$, given the brackets. Does 'simpler' mean something like 'using less ink to write'?

References

Foster, C. (2019). Questions pupils ask: Why can't it be distance *plus* time? *Mathematics in School*, *48*(1), 15–17.

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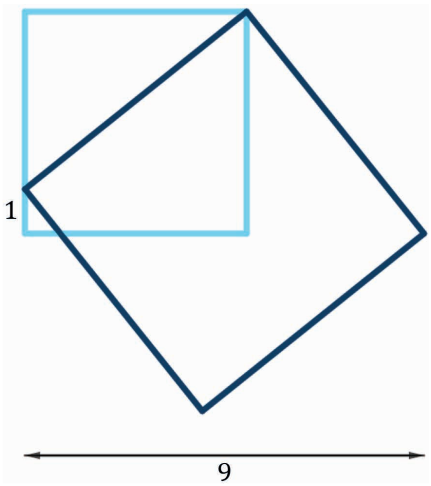
Puzzle 1

by **Paulo Ferro**

The diagram shows two squares. The rightmost vertex of the big square is aligned with the base of the small square. What is the area of the big square?



Solution

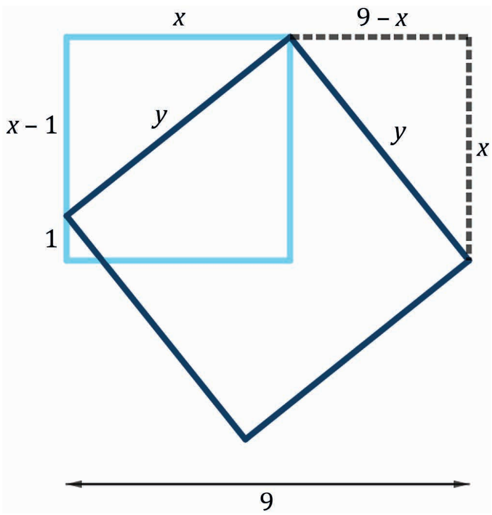
Let x and y be the sides of the small and the big squares, respectively. We can label our diagram as opposite.

Using the Pythagoras' Theorem in the leftmost right triangle:

$$y^2 = x^2 + (x-1)^2$$
$$y^2 = 2x^2 - 2x + 1.$$

Using the Pythagoras' Theorem again but now in the rightmost right triangle:

$$y^2 = x^2 + (9-x)^2$$
$$y^2 = 2x^2 - 18x + 81$$



Then,

$$2x^2 - 2x + 1 = 2x^2 - 18x + 81$$
$$16x = 80$$
$$x = 5.$$

Substituting the value of x in the first equation:

$$y^2 = 2(5)^2 - 2(5) + 1$$
$$y^2 = 41.$$
$$y^2 = 50 - 10 + 1$$

Then, the area of the big square is 41.

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