Questions Pupils Ask!

“What counts as a random number?”

by **Colin Foster**

When I was in the sixth form, my mathematics textbook had a table of random numbers in the back (Note 1). I used to worry about this. The other tables in the back of the book contained *true* things, like trigonometric identities or critical values for the *t* distribution. I enjoyed memorizing things like this and felt a bit ashamed if I ever had to turn to the back of the book to look up these sorts of things. But what about those random numbers? It would be silly to memorize them, wouldn’t it? And yet I knew that people are very bad at inventing random numbers, so when I needed a random number (e.g. for a statistical simulation) I couldn’t just put ‘anything’ off the top of my head, because my ‘anything’ wouldn’t be as random as those numbers in the table. Somehow those numbers were special – so maybe they *were* worthy of being memorized?

After a while, I found that I got to know the first few of these ‘random’ numbers, so they certainly didn’t feel random any more. If random means ‘unpredictable’, then how could these static numbers at the back of the book possibly continue to be random after I had used them so many times? Was I wearing them out? Should I only use them once and then cross them off, like those “one-time pads”, which spies used to use to encrypt their messages – reusing the same set of random numbers would make their messages vulnerable to decoding. This seemed reasonable *within* a particular question, but somehow it felt OK to use the same random numbers again when I moved on to a different question – but what about a different part of the same question? And what if you just needed *one* random number? Was it really worth going to the back of the book to find one, or could you always just use, say, 42. Is 42 a random number? (Can the “Answer to the Ultimate Question of Life, the Universe, and Everything” (Adams, 2017) be a random number?) More recently, I’ve sometimes faced these sorts of questions when using the set.seed function in a language such as *R*. What value should I choose? Is it silly to worry about this?

One response is to say that it doesn’t really mean anything to ask whether a specific number is ‘random’, as though a ‘random number’ were something we could identify by inspection, like a ‘square number’. Perhaps it is only the *process* of generating numbers that may or may not be random, rather than the numbers themselves? Figure 1 shows a set of 100 random numbers between 00 and 99, which my computer produced for me just now.

69	72	88	13	90	69	08	86	47	88
84	17	84	18	89	87	16	52	85	64
43	29	05	22	79	16	02	39	57	33
66	94	84	11	23	31	17	31	16	91
22	14	17	19	01	70	70	94	14	48
42	67	78	11	34	19	21	77	89	96
75	22	00	42	86	98	49	83	23	99
46	14	01	40	91	93	08	59	93	16
12	92	21	06	23	23	60	16	99	58
75	07	17	21	51	59	53	96	40	40

Fig. 1

And Figure 2 shows another set.

42	42	42	42	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42
42	42	42	42	42	42	42	42	42	42

Fig. 2

Now, of course, you don’t believe me about Figure 2. Figure 2 is indeed much ‘less random’ than Figure 1. But doesn’t saying this betray a misconception about randomness? Imagine a large number of companies generating lots of tables of random numbers. Shouldn’t you expect *some* of them to produce the occasional table like this one, just as with lots of lotteries in the world the same sequence of five numbers could come up twice in the same week (Note 2)? So, should Figure 2 be rejected as not random enough or welcomed as something that should be expected very occasionally? If we throw away all the tables that look like Figure 2, are we not really taking randomness seriously enough?

I have found that discussions about randomness can arise at many points throughout the school years (Francome, 2017); for example, when students meet π and ask whether the digits are random (Foster, 2014). How can they be if they are always the same? They feel random if you don't happen to know them, but if you know the first 200 digits of π then Figure 3 doesn't seem any more random than Figure 2.

31	41	59	26	53	58	97	93	23	84
62	64	33	83	27	95	02	88	41	97
16	93	99	37	51	05	82	09	74	94
45	92	30	78	16	40	62	86	20	89
98	62	80	34	82	53	42	11	70	67
98	21	48	08	65	13	28	23	06	64
70	93	84	46	09	55	05	82	23	17
25	35	94	08	12	84	81	11	74	50
28	41	02	70	19	38	52	11	05	55
96	44	62	29	48	95	49	30	38	19

Fig. 3

But if you hadn't spotted this connection with π (say if each pair of digits were reversed, or the whole set were listed backwards), or we started with the 1000th digit of π, or you simply weren't looking that carefully, you would probably have classed the numbers as being as random as those in Figure 1. Does this mean that if I don't want to memorize the random number table at the back of the textbook, and I happen to know quite a few digits of π, I could just use those instead?

I used to do a task with Year 7 classes in which I asked them for homework to throw a coin, and, if it came up heads, throw the coin a further 20 times and write down the *order* of heads and tails obtained; if the initial throw came up tails, they were asked *not* to throw the coin any more, but instead to *make up* a list of 20 heads and tails that they thought looked random. Then next lesson everyone would bring in their lists, with their names on them, and we would pass them around and try to decide whose were genuine and whose were fake (Gelman and Nolan, 2002). It was a lot of fun; for a start, students couldn't copy their homework from someone else! They were generally very bad at guessing, because they would see a run of five heads, say, and think that it was much more unlikely than in fact it is. So, they would assume that the person had made up their data, whereas perhaps they should have assumed the opposite – students were very unlikely to make up unlikely-looking data, whereas real coins don't care!

One time when I set this task, a student came to see me later during the day to say that they had forgotten whether heads meant that they should throw the coin or make up the data, and they wanted to know which way round it was, so that they knew what to do. It made me laugh, because it seemed a completely pointless question, since I simply wanted them to decide at random, so they could just decide either way, couldn't they? (Note 3) Perhaps they would be consciously or unconsciously biased towards tails, as making up the results seems like less work than throwing a coin all those times; however, making up a *really plausible* set of results probably takes quite a bit longer than simply throwing the coin! In class, we would always end up concluding that more than half of the class's data was fabricated, which I generally ascribed to students thinking that it didn't matter whether they threw the coin for real or faked it, and therefore not bothering with all that coin throwing. But maybe it just happened by chance.

Notes

1. You might prefer to read the word 'random' as 'pseudorandom' throughout this article.
2. Indeed, this did happen in 2007 in the North Carolina Cash 5 lottery (Ellenberg, 2014, p. 98).
3. A bit like a student who asks which side of a coin is tails, because they don't know what 'tails' is supposed to look like. While it's useful to know the convention, it of course won't affect any unbiased coin-tossing experiment if you relabel heads as tails and tails as heads.

References

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