Questions Pupils Ask!

What is the net for a parallelepiped?

by **Colin Foster**

Visuospatial problems can be a great leveller in mathematics. There are people with fancy mathematics degrees who find it very difficult to tell whether two three-dimensional solids sketched from different angles are the same or different, and yet there are school pupils who can do this sort of thing just by staring – and think that it's obvious! So when these sorts of problems arise in the classroom the gap between what the pupil can do and what the teacher can do can be quite small, if it exists at all!

One of my favourite bits of project work with Year 8 pupils is to build a shape sorter – those toys that toddlers play with, where each solid fits through one and only one of the holes (Note 1). Designing a shape sorter that does that can be quite challenging, as it's very important that no shape will go through the wrong hole – we don't want to confuse those poor little toddlers! Typically, pupils go for things like a cube, a non-cube cuboid, a cylinder, a right-angled triangular prism and an equilateral triangular prism. An extra constraint can be that all of the solids must fit inside the container (e.g. a shoe box) at once. All sorts of interesting questions can arise, such as "Can you have more than one cylinder?" or "Can you make a shape sorter just using cuboids?" I have found that more adventurous pupils tend to go for many-sided prisms or pyramids, such as hexagonal-based prisms, which tend to be fiddly to construct but not conceptually much more difficult. So perhaps a more interesting direction to go in is to make a non-cuboid parallelepiped.

Pupils always think that 'parallelepiped' is a very funny name, and I often wonder if I am saying it correctly! A parallelepiped (Fig. 1) is a prism whose faces are all parallelograms. It includes the special case of a cuboid (and thus a cube), but in general the parallelograms do not need to be rectangles. These shapes don't come up very much in school until the sixth form, when some students will meet them when working with 3 by 3 matrices and the scalar triple product. Sometimes with younger pupils I have pushed a stack of paper sideways to illustrate shearing of a rectangle into a parallelogram with preservation of area, but the focus here is on the parallelogram face rather than the parallelepiped as a whole. So how do you make a net for a parallelepiped? If you've never tried to draw a net for a parallelepiped, you might like to explore that now, before (or instead of) reading the rest of this article!

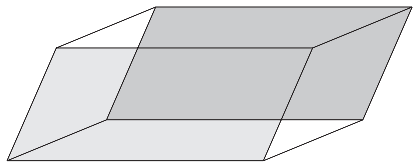

Fig. 1 A parallelepiped

What usually happens is that pupils begin by drawing the front parallelogram (shaded light grey in Figure 1) and then pause for a while, wondering whether the parallelogram on the top surface can/must be congruent to the first parallelogram, or whether its angles need to be different. Pupils attempting this will already know that a cuboid net can be made as shown in Figure 2, so they tend to try to adapt this by turning

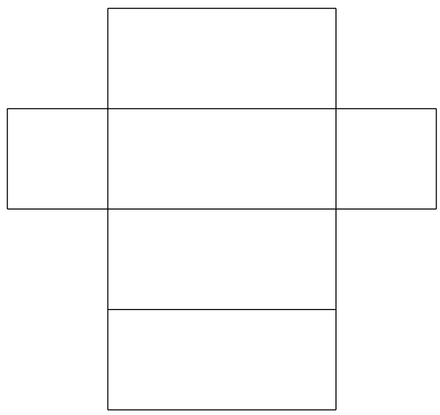

Fig. 2 Net of a square cuboid

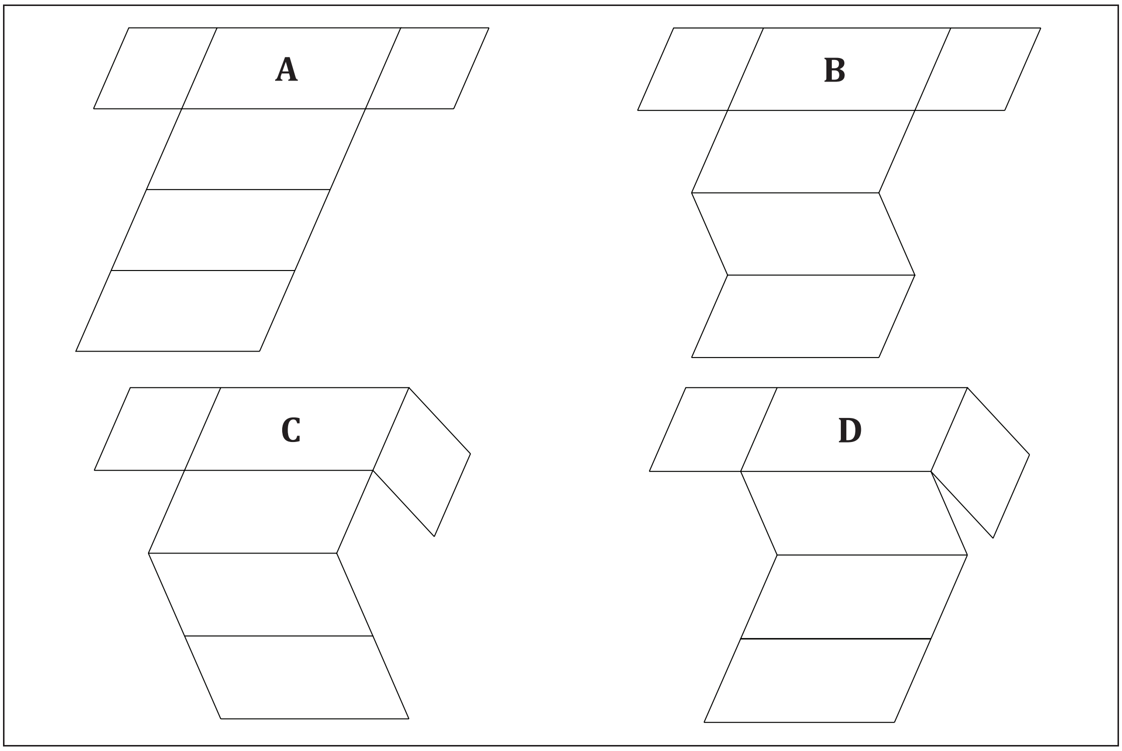

Fig. 3 Do any of these fold up to make a parallelepiped?

the rectangles into parallelograms, which is a sensible way to start. (Isometric paper can be useful.) Figure 3 shows some attempts – do any of them fold up to make a parallelepiped? Why or why not? (Note 2)

One way to deal with this (if you're totally desperate) is to cut out the separate parallelograms that you want to have on your finished parallelepiped, fit them together with sticky tape to make the parallelepiped, and then cut it open along some edges and look at what you have got! But that is very much a last resort if you can't get there any other way!

The crucial observation that helps is that opposite faces, when viewed from the outside, must be mirror images (Note 3). This may be counterintuitive, because when we look at the parallelepiped from one direction, as in Figure 1, we see both shaded parallelograms on top of each other, and the same way round. But when you unfold the net and lay it flat, one of them has to turn over, so you see the *reverse* side of one of these, and that is why they have to be mirror images when drawn on the net. In a cuboid, this reflecting makes no difference, because a rectangle is its own reflection, but this is not true for a general parallelogram. So, in Figure 4, a rotation of 180° will *not* transform one of these parallelograms into the other – and this may be surprising to pupils. This is an important insight, but there are other aspects to consider as well to get a working parallelepiped net. Can you find all the possible nets, or state necessary and sufficient criteria for a net for a parallelepiped? (Note 4)

For a free lesson plan relating to this lesson, see Foster (2017).

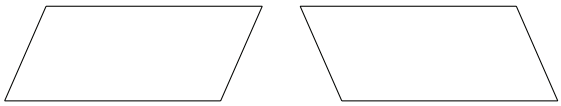

Fig. 4 A general parallelogram and its mirror image

Notes

1. I first heard of the idea from Keith Proffitt.
2. The way in which some of these images jump out of the page as three-dimensional can interfere with the visualization necessary to imagine them folded up.
3. In general, the faces of a parallelogram are chiral (non-superimposable mirror images), even though the entire parallelepiped is not.
4. A lovely website for producing nets of a parallelepiped is available at: **www.templatemaker.nl/index.php?template=parallelepiped&lang**

Reference

Foster, C. 2017 'A Fitting Challenge', *Teach Secondary*, **6**, 6, pp. 48–49. Available free at **https://www.teachwire.net/news/ks3-maths-lesson-plan-promote-understanding-of-3d-shapes-by-making-a-childr**

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