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BOOK REVIEWS

Teaching with tasks for effective mathematics learning, by Peter Sullivan, Doug Clarke and Barbara Clarke, New York, Springer, 2013, 197 pp., £90 (hardback), ISBN 978-1461446804

There has recently been increased interest in the research and development of task design within mathematics education. This follows the production over time of theoretically-informed materials and approaches such as those from Realistic Mathematics Education (De Lange, 1996), Connected Mathematics (Lappan, Fitzgerald, Feyl, Friel, & Phillips, 2009) and the Centre for Research in Mathematics Education at the University of Nottingham (also known as the Shell Centre) (Swan, 2006). In this book, Sullivan, Clarke and Clarke make an important contribution to the growing literature on mathematics tasks as they outline the findings from their three-year project *Task Types in Mathematics Learning*, in which they worked with grade 5–8 teachers from three varied clusters of primary and secondary schools in Victoria, Australia. They describe a wide variety of different mathematical tasks and their observed effects in the classroom, how teachers' pedagogical practices were informed through the process, and insights from the students. They also report on a subset of their teachers who took responsibility for designing and teaching extended sequences of lessons.

All of the authors' research is "based on an assumption that choice of tasks and the associated pedagogies are key aspects of teaching and learning mathematics" (p. 14) and that "the nature of teaching and what students learn are defined largely by the tasks that form the basis of their actions" (p. 57). In line with other researchers in this area, they see tasks as fundamental to all mathematics classroom activity. The authors "use the term *task* to refer to information that serves as the prompt for student work" (p. 13), and the study focuses on three *task types*:

- *Purposeful representational tasks*: These are tasks that use "models, tools, and representations" (p. 23) in a conceptually (rather than procedurally) focused manner to develop students' understanding. One example task given employs a fraction wall to solve problems involving operations with fractions.
- *Contextualised tasks*: These are tasks "built around practical (or 'real') contexts" (p. 39) which engage students in purposeful mathematics. One example given is a task in which students are shown a photograph of a signpost showing distances to several major cities across the world and asked where the sign might have been photographed.
- *Content-specific open-ended tasks*: These are tasks with "more than one (preferably many more than one) possible response" (p. 57), but which are

> tightly focused on a particular area of content in the conventional mathematics curriculum. One example given is a task in which students are told that someone wrote a sentence containing only five words, that the mean number of letters in each word was 4 and that none of the words had four letters, and are asked what the sentence might have been (p. 183). (One student response was: "Jam is an acquired taste" (p. 184)).

Descriptions of exemplar tasks are followed by detailed accounts of what took place when they were used in the classroom, including rich descriptions of students' responses, making this a practical book equally useful to the classroom teacher and to the academic researcher.

The book is structured around Stein, Grover, and Henningsen's (1996) widely-utilised conceptual framework in which a mathematical task is seen as living through three stages: (1) as in the curricular/instructional materials; (2) as set up by the teacher in the classroom; and (3) as implemented by the students in the classroom. After three preliminary chapters, in which the authors outline and justify the project, the rationale for their focus on mathematical tasks and the role of teacher knowledge in effective task use, they devote a chapter each to the three task types listed above, justifying their use, detailing multiple examples and summarising the outcomes in the classroom and the teachers' reactions. These chapters form the basis for drawing out important generalisations regarding each particular task type. The authors then move on to consider how tasks function within lessons, and lessons within sequences of lessons. A subset of the teachers from the study volunteered to work with the researchers to develop lesson sequences around volume and capacity, ratio and proportion, interpreting data and financial literacy. The quality of these lesson designs is testimony to the creative potential of collaborative groups of classroom teachers and a suitable retort to those who would question teachers' capabilities in this regard (Wittmann, 1998). In some particularly interesting later chapters, the authors focus on students' perceptions and preferences, and in the final chapter (which is virtually an appendix), there is a collection of numerous rich tasks, exemplifying the three categories. For each task given, the authors provide an account of its mathematical focus, some insights gained when teaching it, potential student difficulties, sample student work, suggested enabling and extending prompts and ideas about 'the next lesson'.

I immediately found myself engaging enthusiastically with the tasks presented. Inevitably some were familiar, but many were new to me, or seemed to be significantly-enhanced versions of well-known canonical tasks. For example, in "Money measurement" (p. 190), students are told that they have won a prize and may choose one of:

- one metre of $2 coins (lying flat);
- one square metre of 5-cent pieces (edges touching, lying flat);
- a 1 litre milk carton full of 20 cent pieces; or
- one kilogram of $1 coins.

This has all the signs of being a carefully-crafted design with many rich possibilities for students, and certainly a very worthwhile basis for a lesson or sequence of lessons. This task was characterised as a content-specific open-ended task, and I frequently

found myself liking the tasks and appreciating the descriptive and analytical work but questioning the categorisations. Clearly, there are many different assumptions and methods that students might use in tackling this problem, but are there really multiple 'correct' answers? For me, the openness in this task is in the process rather than in the answer – *open-fronted* in the assumptions that go into defining the problem, and *open-middled* in the choices of approach made in solving it, but not particularly open-ended in the outcome.

The authors acknowledge that the categorisations of task type are not watertight, conceding, for instance, that purposeful representational tasks, while not being contextualised, may sometimes have a 'hook' that helps to engage the students (p. 36). Likewise, the contextualised tasks exhibited appeared to me frequently to have an open-ended element. However, the authors do not claim that this triad of task types is non-overlapping or exhaustive; that it is the beginning of a much longer list is tacitly acknowledged in the authors' statement that they abandoned their initial plan to include "extended investigations" as a fourth task type because teachers felt that they did not have enough lesson time in their crowded curricula to do this (p. 138). The authors also explain that they chose not to focus on tasks that develop fluency, "since such tasks are well represented in every school mathematics text we have seen" (p. 8). However, while acknowledging that this is true, I have argued elsewhere for the need to find ways of embedding practice of important mathematical procedures within richer and more authentically mathematical contexts (Foster, 2013).

The book contains a wealth of helpful guidance on lesson planning, such as the value of asking teachers "What would the next lesson look like?" as a way of promoting a more extended perspective over a sequence of interrelated lessons (p. 81). The authors also offer valuable advice concerning the construction of contextualised tasks, by starting with the interests of students, taking a particular event from the mass media, and taking digital photographs of objects or places with mathematical potential. These tactics resonate with the mathematical modelling approach taken by Dan Meyer (http://blog.mrmeyer.com/) and websites such as www.problempictures.co.uk and www.mathematicalbeginnings.com. Using photographs and videos of real-world objects would seem to present a possible way to avoid the ever-present danger (which the authors acknowledge) of pseudo-real-life problems, which risk trivialising both the scenario and the mathematics. Frank acknowledgements are given regarding possible weaknesses in the different task types, such as that purposeful representational tasks seem much better suited to number than to chance and data (p. 32). For me, the most powerful task type presented is the content-specific open-ended one, which accommodates tasks that are "accessible by students, able to be used readily by teachers, foster a range of mathematical actions, and contribute to some of the important goals of learning mathematics" (p. 57). Such rich tasks would seem to offer students deep opportunities for learning mathematics.

The students in this study were adept at distinguishing between tasks that they enjoyed and those that they learned from. Some of the conclusions drawn from the students' comments were extremely stimulating, such as the finding that nearly half of them "in describing their 'best' mathematics lesson referred to specific content" rather than to a general type of lesson task (p. 113). Overall, no one task type was significantly more popular with the students than the others. "Each of the task types

was liked most by some students, and likewise each of the types is rated as the one from which they can most learn'' (p. 106). The complicated balance of answers may indicate that above all students appreciate variety in their learning of mathematics. The authors were surprised that the students who rated themselves as the most confident were more likely to choose tasks that were perceived to be easier (p. 104), but Dweck's research (2000) on mind-sets suggests that high-performing students with a fixed mind-set have a lot to lose by opening themselves up to challenging tasks, and are likely to be risk-averse in precisely this way. However, many students did want to learn by being challenged (p. 143). The authors expanded their collection of student data during the project, due to the ''useful insights it provided'' (p. 136). In many cases the students' detailed, mature and articulate comments were profound, and the authors comment that they ''are possibly as helpful a source of advice for teachers as are teacher educators'' (p. 120)!

This book underlines the value of research-designed and collaboratively-developed mathematics tasks. Realism pervades the pages of the book: although the authors are visionary regarding what tasks can achieve, they caution that there are few tasks that can meet all the criteria that they list, and urge teachers to ''create a balanced and healthy diet, even though the individual 'meals' may include only a few of these ingredients'' (p. 21). The authors' research highlights ''the importance of providing a wide range of types of tasks in a variety of different sequences, in order to meet the needs of as many students as possible'' (p. 134). The tasks studied:

> allow students opportunity to have a sense of control by allowing them to make decisions, are interesting to the students, incorporate a rationale for them to engage, provide some challenge, reduce the risk of failure, and for which success provides the motivation for further engagement. (p. 10)

This account of the lives of these mathematics tasks makes inspiring reading, and I can only hope that this book will be very widely read, and that the tasks found here, and others like them, will be able to benefit many more students of mathematics.

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LOST Opportunities: learning in out-of-school time, edited by B. Bevan, P. Bell, R. Stevens and A. Razfar, London, Springer, 2012, 297 pp., £90 (hardback), ISBN 978-94-007-4304-5

LOST Opportunities is a volume of work focusing on the learning of mathematics and science that takes place outside of school. Its publication followed a series of meetings organised by the Center for Informal Learning and Schools, based at the Exploratorium, San Francisco. The book is in three parts. Part 1 addresses the question "What counts as Math and Science?" Part 2 explores how people learn across settings, and asks how better understanding of this learning can inform educational equity strategies. Part 3 focuses on learning that occurs in organised, but still out-of-school, settings, such as after-school clubs, summer camps and museums, and takes a wider view of learning content to include other aspects of Science, Technology, Engineering and Mathematics (STEM).

The variety of discussion in *LOST Opportunities* is interesting, and represents an important issue raised throughout the book, namely that research in out-of-school learning is still developing in terms of agreement on constructs and key points of focus. The variety in the book comprises at least three dimensions: the learner; the setting; and, the support. Learners are sometimes children of various ages and sometimes adults. The setting ranges from those that are formal and school-like to those that are informal and home-like. The learning that is in focus is sometimes learner-directed and self-supported, and sometimes peer-directed, parent-directed or instructor-directed. This variety, for me, is linked with the fact that the content of the book is defined in the subtitle in a negative way: "Learning in Out-of-School Time". It is difficult to know how the content could be defined in a more positive way given that the phenomena being researched are so diverse. For some readers, this apparent lack of focus may be a major shortcoming, but on the other hand, it is indicative of the fact that research on out-of-school learning is very much needed. Indeed, part of the rationale for *LOST Opportunities* is the scarcity of research on the learning of mathematics and science outside of school. There are several challenges involved in the study of learning out-of-school, which explains this scarcity to some extent. One is that it is very much harder for the researcher to access learning out-of-school than learning that occurs in school. Schools are clearly an extremely useful resource for a researcher who is interested in learning; children in schools are a relatively captive participant group, who are engaged by law in learning activity. Compared with