# Plus-Minus Graphs

**Colin Foster**

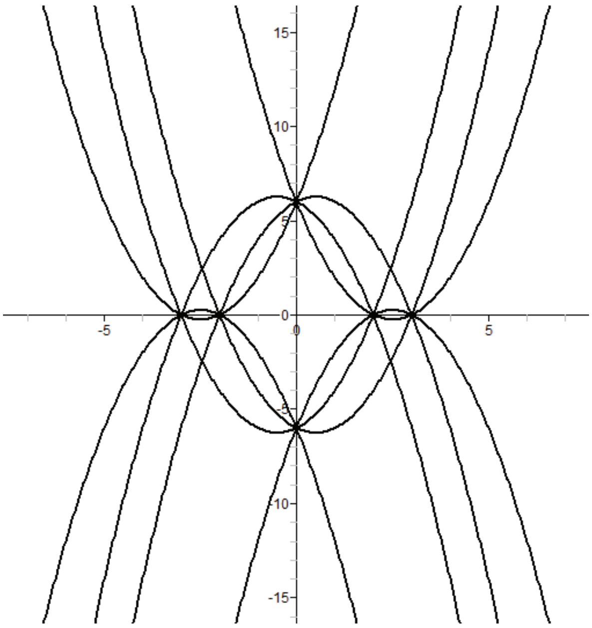

Figure 1: A family of curves

Look at the curves shown in Figure 1. What can you deduce about them? Can you find their equations? Can you see how they are connected? You might like to ponder this before reading on.

Were you able to distinguish the separate curves? It may help to know that they are all parabolas. They were drawn using graph-drawing software by just entering one fairly simple ‘equation’.

The equation for the curves is $y = \pm(x \pm 2)(x \pm 3)$, leading to the eight parabolas shown. They have a very pleasing symmetry. Would you expect the same result if you expanded the brackets first and entered $y = \pm\ x^2 \pm 5x \pm 6$?

In fact, the expanded version $y = \pm x^2 \pm 5x \pm 6$ leads to a completely different-looking graph (Figure 2), and you might like to think about why.

Since each ‘plus or minus’ sign has two possibilities, the family $y = \pm x^2 \pm 5x \pm 6$ consists of the following eight curves:

$y = x^2 + 5x + 6 = (x + 2)(x + 3)$

$y = x^2 + 5x - 6 = (x + 6)(x - 1)$

$y = x^2 - 5x + 6 = (x - 2)(x - 3)$

$y = x^2 - 5x - 6 = (x - 6)(x + 1)$

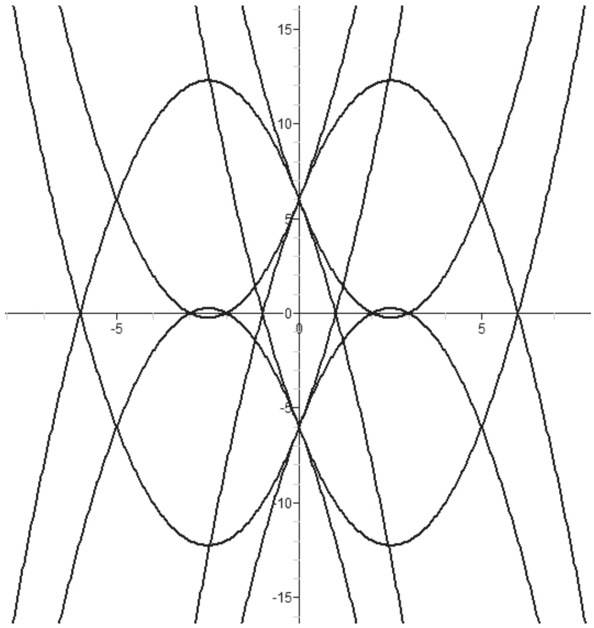

Figure 2: The family of $y = \pm x^2 \pm 5x \pm 6$

$y = x^2 + 5x + 6 = (-x + 6)(x + 1)$

$y = -x^2 + 5x - 6 = (-x + 3)(x - 2)$

$y = -x^2 - 5x + 6 = (-x + 1)(x + 6)$

$y = -x^2 - 5x - 6 = (x + 2)(x + 3)$

It is quite simple to produce some beautifully intricate drawings by taking familiar curves and ‘plus–minusing’ them (Figures 3 and 4).

This all began when I took my sixth form class into the computer room and asked them to draw some graphs ‘different from any graphs you have ever drawn before’. All sorts of things appeared, but the presence of ± on the menu at the top of the screen prompted one student to enter $x = \pm 1 \pm 1 \pm 1 \pm 1$. He was surprised at the result and began to explore what happened with similar equations. Can you predict how the following will look?

$x = \pm 1 \pm 2 \pm 3 \pm 4$

$x = \pm 1 \pm 3 \pm 5 \pm 7$

$x = \pm 2 \pm 3 \pm 5 \pm 7$

$x = \pm 1 \pm 2 \pm 4 \pm 8$

$x = \pm 2 \pm 4 \pm 6 \pm 8$

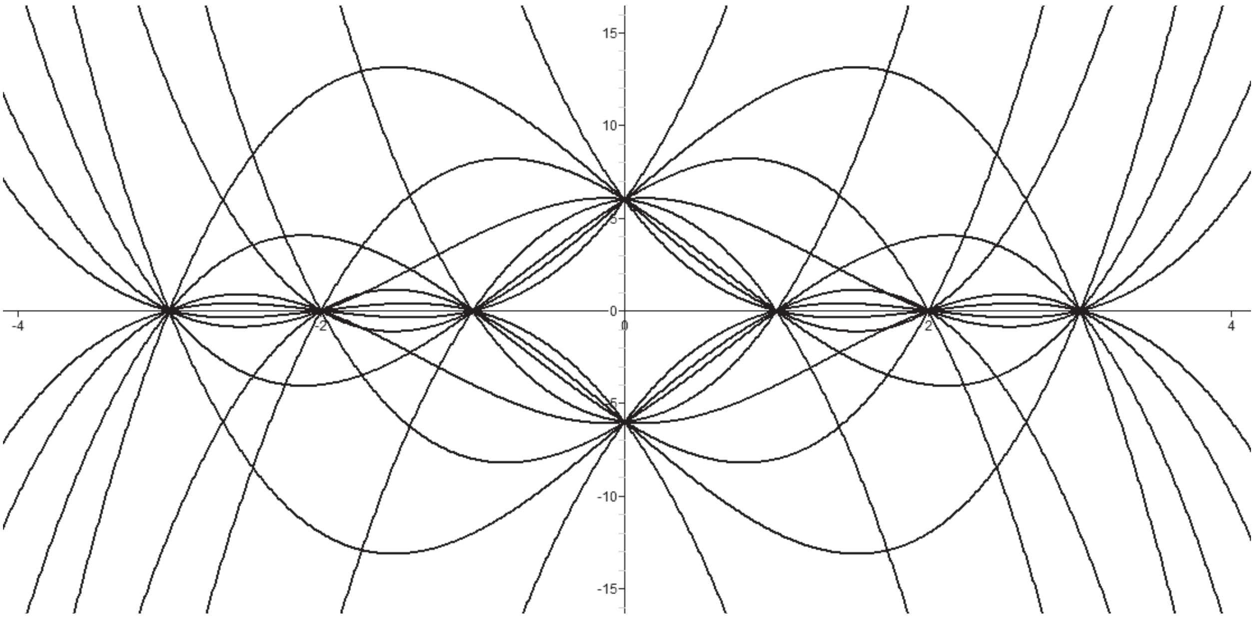


Figure 3: The family $y = \pm(x \pm 1)(x \pm 2)(x \pm 3)$

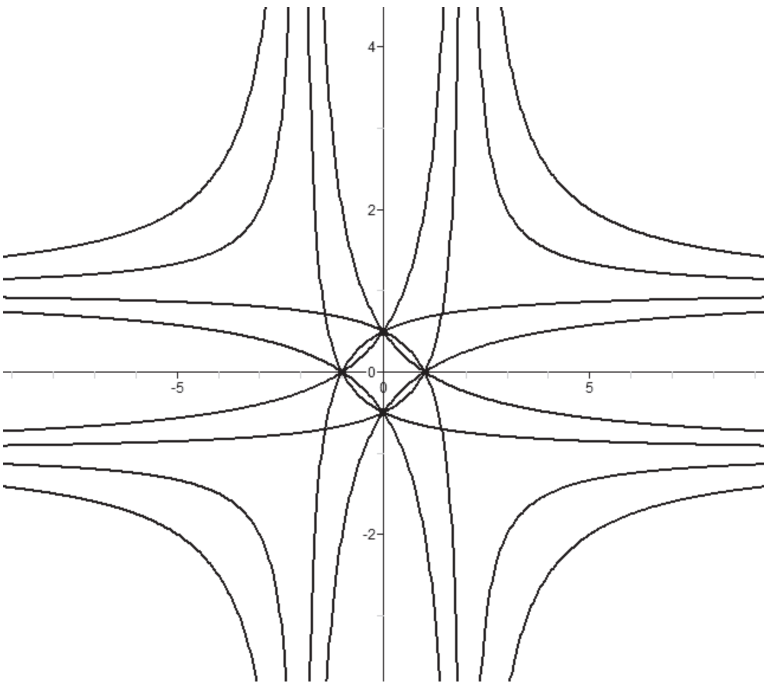


Figure 4: The family $y = \pm\left(\dfrac{x \pm 1}{x \pm 2}\right)$

What happens if you change the order of the terms? Why?

How many lines in total will you get for each one? Why?

Will they be symmetrical about the $y$-axis? Why/why not?

What will be their positions? Where will the outer (end) lines be? Why?

Will they be evenly spaced? When? Why?


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