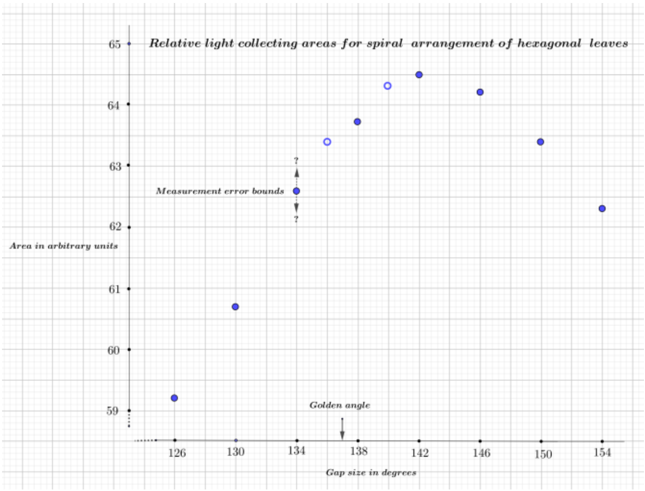
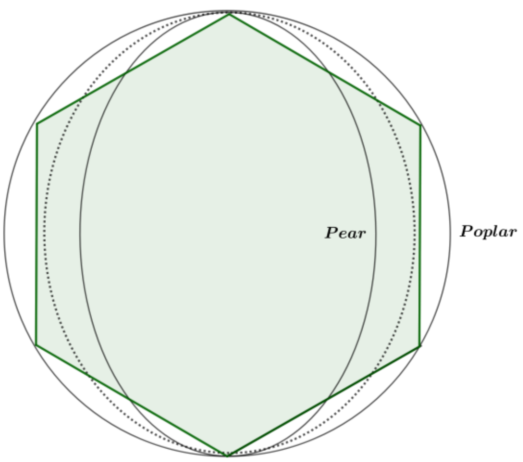
Here is my plot, with much uncertainty about the size of the error in my area measurements. I have taken readings every $4°$ (the filled in circles) with a couple of extra ones around the golden angle (the open circles).



For 12 leaves of our hexagon plant, we seem to have a peak at $142 \pm 1°$. As far as the arrangement of leaves (*phyllotaxis*) goes, this puts it in the same group as the poplar, which has a more or less circular leaf, and the pear, whose leaf is an ellipse with approximate aspect ratio $3 : 2$.



Bearing in mind that the area of an ellipse is just πab, *where* a *is half the long axis and* b *half the short one, confirm that the hexagon leaf area falls within* 1% *of the mean of the leaf areas of the poplar and the pear.*

Find a plant with a very different leaf shape and try to model it in the way described. (Though you can measure the areas of circles and ellipses with GeoGebra, to measure the area of overlapping shapes, you have to approximate them as polygons.)

Paul Stephenson

FACTOR PUZZLES

Factor puzzles (for want of a better name) are a nice example of an inverse process being (much) harder than the direct process (Note 1). First, let us do the direct process: choose four positive integers and write them in some arrangement on adjacent sides of a 2×2 square:

	3	8
4		
6		

Then fill in the four products in the four cells:

	3	8
4	12	32
6	18	48

This is pretty straightforward multiplication tables practice. But suppose, instead, you had the inverse problem – you were just given the *interior numbers*?

12	32
18	48

Could you have worked out the four outside numbers that must have been used? This *inverse process* is much harder than the direct process. And can you be sure whether there might be more than one possibility?

A systematic approach is to consider the common factors of each row and column, written below in italics:

	1, 2, 3, 6	*1, 2, 4, 8, 16*
1, 2, 4	12	32
1, 2, 3, 6	18	48

Then choose the row or column with the *smallest* number of common factors (i.e., row 1 above) and try each of these factors as an outside number (in bold below):

	12	
1	12	32
X	18	48

(i)

	6	16
2	12	32
3	18	48

(ii)

	3	8
4	12	32
6	18	48

(iii)

In **(i)**, trying the factor 1, we obtain 12 as an outside number, and then find that row 2 is impossible to complete, since 12 is not a factor of 18. However, we find that both 2 **(ii)** and 4 **(iii)** work as numbers at the top of the left-hand side, and so we obtain two solutions – the one we started with **(iii)** and another one **(ii)**. Students may be surprised that there can be more than one solution.

At this point there are many questions we might pose:

1. What other approaches are good for solving these?
2. When do we get *more than one* solution?
3. When do we get *no* solutions?

These puzzles are easy to invent, but the challenge becomes not just finding 'a' solution but being sure how many solutions there are. Students might be challenged to invent puzzles with exactly two solutions or three solutions. What is the *maximum* number of solutions possible?

It may not be immediately apparent, but these diagrams are the same in structure as what are sometimes called 'ratio tables'. If we generalise the four outside numbers to p, q, r and s, then we have:

	p	q
r	pr	qr
s	ps	qs

Now we can see that the entries in the second column are $\frac{q}{p}$ times those in the first column, and the entries in the second row are $\frac{s}{r}$ times those in the first row, just as in any ratio table. We can also see that the 'determinant' must be zero, since the product of the entries on the main diagonal – $pr \times qs$ – has to be equal to the product of the entries on the other (anti-) diagonal – $ps \times qr$. This means that there is definitely no solution to a factor puzzle if these two diagonal products are not equal, and it turns out that if they *are* equal then there is definitely at least one solution (Abusaris, & Alhami, 2022).

There are lots to think about with factor puzzles, and it is possible to extend the idea to 2×3, 3×3 or other sizes of puzzle, as well as to include fractions, decimals and even algebraic expressions in the cells. For example, how would you go about solving this puzzle?

$\frac{1}{6}$	$\frac{2}{9}$	$\frac{1}{2}$
$\frac{3}{20}$	$\frac{1}{5}$	$\frac{9}{20}$

Note

1. These are related to 'multiplication table puzzles' like https://findthefactors.com/solve-find-the-factors/.

Reference

Abusaris, R., & Alhami, K. (2022). Factor puzzles from definition to applications [pre-print]. *F1000Research 2022*, *11*, 727. https://doi.org/10.12688/f1000research.111241.1

Colin Foster

QUICKIE 42B

My gross brother also only owns pairs of black and white socks. He has 33 pairs of black socks and some pairs of white socks.

If he draws two socks from his sock cupboard at random, the probability they match is also ½.

How many socks does he own?