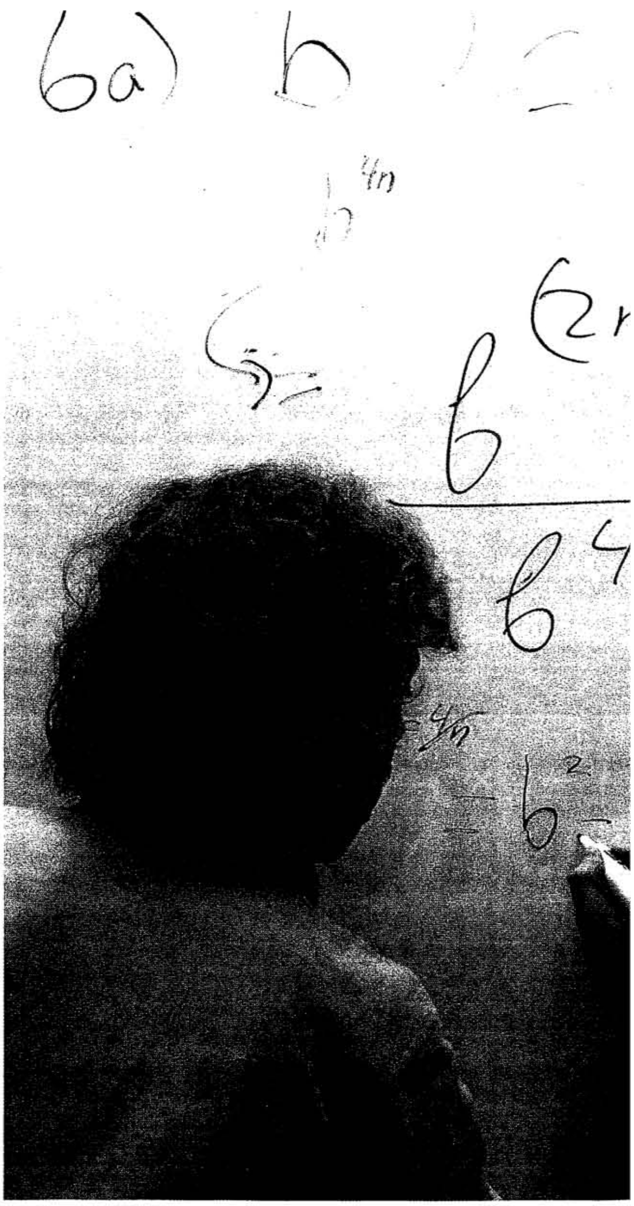
Solving the

■ maths

Algebra proving to be a real puzzler? **Colin Foster** says the answer is to put a pupil's name to the letter



In algebra, it's natural to think of letters as objects – things you have to move around on the page according to certain rules. Whether this counts as a misconception or not is a moot point, but once pupils begin asking, "Can I put this X here?" or "Is it OK to move the Ys so they're next to each other?", it is apparent that the meaning of the letters as generalised numbers has to some extent been lost. Pupils are seeking rules to follow so they no longer have to think about the meaning of what they are writing.

Lessons on simplifying, expanding, factorising or handling indices can descend to the point where pupils keep seeking reassurance from the teacher that what they are doing is OK: only the teacher can say whether something makes mathematical sense or not. This makes maths feel like a foreign language and pages of errors such as $2a^4 \text{x} 3a^6 = 5a^{24}$ are readily produced by only a slight misapplication of rules.

To address this problem, pupils need to be encouraged continually to see letters as standing for numbers. This is the basic idea of algebra, but seems to get lost at times amid rules for pushing symbols around on the page. As the teacher, I don't want to be the authority who decides whether something is correct or not, otherwise I'm the only person in the room doing any maths. I want to place that responsibility back on the pupils.

One strategy to offer is substitution. By substituting any number (for example, an easy one like 10) for the letter A in the expression above reveals

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X factor

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'Reinforce at every stage the idea of letters as generalised numbers instead of some weird symbol'

that it is false – it doesn't say something true about how numbers work, therefore it is not right. It's wrong because it doesn't work, rather than because the teacher doesn't like it.

With some of my classes, this has developed, over a period of time, into a view of algebra that a letter A, for instance, can be thought of as the number belonging to someone whose name begins with an A, Anupe, for instance.

So with some of my classes, whenever it is convenient to do so, we find ourselves talking about David's number, because the problem happens to have a D in it. (If you have a Xanthe in your class, you are obviously very lucky – otherwise someone can volunteer to be Xanthe for the day).

The benefit of this approach is that it reinforces at every stage the idea of letters as generalised numbers. Instead of some weird symbol, x is just 10 or -3 or whatever Xanthe decided it was going to be. So we can keep checking as we go that "it still works for Xanthe's number". This is not really a one-off task: it is more of a pervasive way of thinking about algebra.

When pupils create their own equations for others to solve (or do this in pairs for simultaneous equations), they can easily use the initial letters of their own names – I have done this for many years, but only recently have I realised the value of keeping this going when pupils are trying to simplify such things as $6b-2b$.

It isn't "six Hs and then we take away two of them" (thinking of letters as objects); it's "Six lots of Harnam's number minus two lots of Harnam's number", and we can check it works if Harnam picks a number to try it with ("Does that make what you got, Harnam? Does it still work if he picks a negative number?") ■

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Resources

Book John Mason with Alan Graham and Sue Johnston-Wilder (2005) *Developing Thinking in Algebra*, Open University (Paul Chapman Educational Publishing, £19.99).

Websites www.bbc.co.uk/education/mathsfile/gameswheel.html and click on Late Delivery
■ www.waldomaths.com and click on 11-16 and then equations
■ http://nlvm.usu.edu

Figuring it out

■ When asked, "Is this right?" try to bounce the responsibility back to the pupil for checking, not by looking the answer up or asking someone else (which is no better than asking you), but by substituting an easy value to see if it works. "It's right if it's true" and "Have you checked to see if it works?" can be good responses.
■ Be happy for pupils to answer fewer questions but be confident that they are correct, rather than race through pages of work without much sense of what it means or whether the teacher is going to tick it or cross it. (There is a case for slowing down and thinking when doing "easy" questions like "simplify", otherwise they will later on, just as easily and logically, write $3x+4x^2=7x^3$)
■ Ask pupils to put a symbol, such as traffic lights, next to each answer describing how confident they are it is correct. This can lead to a discussion of why.
■ Take opportunities to make sense of algebraic expressions (for example, deciding whether two expressions are equivalent) by trying a number suggested by a pupil whose name begins with the letter in question.