[MATHS PROBLEM]

A CLEAR ACCOUNT OF EVENTS

Colin Foster looks at how students can confuse 'independent events' with 'mutually exclusive' events when studying probability

In this lesson, students clarify the distinction between these by trying to devise examples of events for which one, both or neither apply.

THE DIFFICULTY

Start by showing students these four quantities relating to events A and B:

$p(A)$ $p(B)$ $p(A \cup B)$ $p(A \cap B)$

What does each of these mean? How can you best explain them in words?

Students might respond that '$\cup$ means or' and '$\cap$ means and', but other students might use 'and' to describe $A \cup B$, so clearer language is needed. $p(A \cup B)$ is the probability of either A or B (*or both*), whereas $p(A \cap B)$ is the probability of both A and B.

What do you understand by 'independent' events?
What do you understand by 'mutually exclusive' events?
Can you express these in symbols?

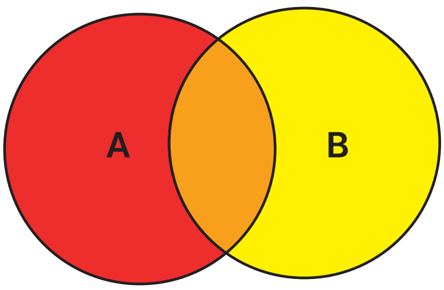
Students could talk in pairs. Many students may be confused about these terms and how they differ.

THE SOLUTION

Mutually exclusive events cannot *both* happen – they are incompatible alternatives, so, if A and B are mutually exclusive, then $p(A \cap B) = 0$.

If two events A and B are *independent*, then $p(A)$ doesn't depend on whether B happens or doesn't happen, and $p(B)$ doesn't depend on whether A happens or doesn't happen. This means that $p(A \cap B) = p(A).p(B)$.

These terms can be quite abstract, and drawings, such as Venn diagrams, can help students to see what they really mean.



$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

Now look at this table (an electronic version of this with the answers is available at bit.ly/ts116-mp1). Fill in the missing probabilities on each row and decide where ticks should go in the right-hand three columns.

#	$p(A)$	$p(B)$	$p(A \cup B)$	$p(A \cap B)$	*Mutually exclusive?*	*Independent?*	*Impossible?*
1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$				
2	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$				
3	$\frac{1}{2}$	$\frac{1}{3}$	1				

Watch out for students thinking that mutually exclusive means that $p(A) + p(B) = 1$ (i.e., *mutually exhaustive*).

Checking for understanding

To assess students' understanding, ask them to make up their own puzzle table like this one, with some numbers and ticks on each row.

Colin Foster (@colinfoster77) is a Reader in Mathematics Education in the Department of Mathematics Education at Loughborough University. He has written many books and articles for mathematics teachers. foster77.co.uk, blog.foster77.co.uk