[MATHS PROBLEM]

ADDING AND SUBTRACTING NUMBERS IN STANDARD FORM

Numbers written in standard form can be confusing for students to add or subtract, says **Colin Foster**

In this lesson, students explore alternative ways of adding and subtracting numbers written in standard form

THE DIFFICULTY

Which of these numbers is in standard form and which isn't? How can you tell?

2.1×10^4 and 37×10^4

The first number is in standard form, because it's of the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer. The second number ***isn't*** in standard form, because 37 > 10.

How would you write 37×10^4 in standard form?

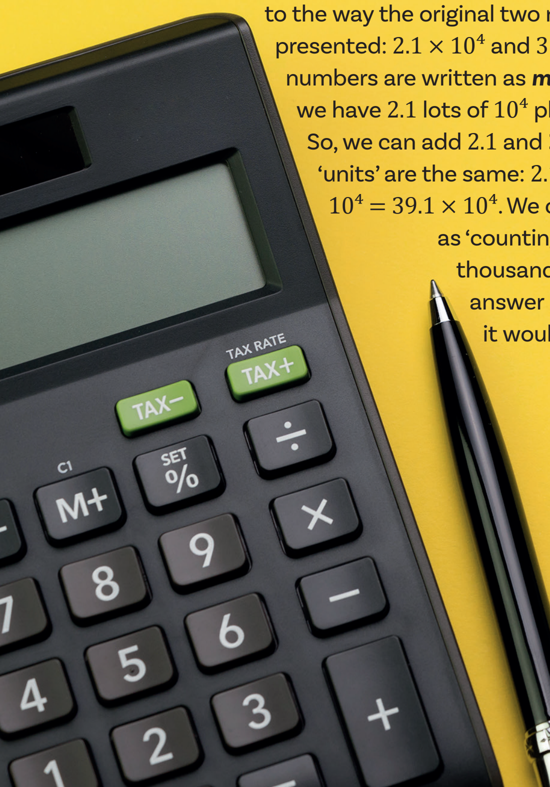
In standard form, this number would be 3.7×10^5. The 37 has been ***divided*** by 10 and the 10^4 has been ***multiplied*** by 10, making 10^5, so the ***product*** is unchanged. We can imagine a factor of 10 leaving the 37 to join the 10^4.

*How would you **add up** these two numbers?*

Students may be unsure, or they may suggest wrong answers, such as 5.8×10^5 or 5.8×10^{10}.

THE SOLUTION

It's easier to see the answer to this question if we go back to the way the original two numbers were presented: 2.1×10^4 and 37×10^4. Both numbers are written as ***multiples of 10^4***: we have 2.1 lots of 10^4 plus 37 lots of 10^4. So, we can add 2.1 and 37, because the 'units' are the same: $2.1 \times 10^4 + 37 \times 10^4 = 39.1 \times 10^4$. We can think of this as 'counting in ten thousands'. Giving the answer in standard form, it would be 3.91×10^5.



How could you do this by 'counting in hundred thousands' instead of by 'counting in ten thousands'?

We could write both numbers as multiples of 10^5:

$2.1 \times 10^4 + 3.7 \times 10^5 = 0.21 \times 10^5 + 3.7 \times 10^5 = 3.91 \times 10^5$.

Get students to explain what's happened here.

Which way do you think is easier? Why?

*Make up five examples of additions like this. Solve them in **both ways** (i.e. by making the powers of 10 match in **two different ways**, like this).*

*Students may see an analogy with finding **common denominators** when adding fractions. In both cases, we need to find a 'common unit' before we can add.*

*How would you work out the **difference** between the two numbers that we started with?*

The process is almost identical. To find the difference, we first need to decide which number is larger, and then subtract the smaller number from this: 37×10^4 is greater than 2.1×10^4, so we calculate $37 \times 10^4 - 2.1 \times 10^4 = 34.9 \times 10^4 = 3.49 \times 10^5$. Alternatively, in 10^5s, we have $3.7 \times 10^5 - 0.21 \times 10^5 = 3.49 \times 10^5$.

Checking for understanding

*How would you find the **sum** and the **difference** of these two numbers?*

2.1×10^3 and 37×10^5

What is different this time? The method is the same here, but the numbers happen to be further apart in size. This time, we can choose to work in units of 10^3, units of 10^5 or units of 10^6. However we do it, the sum comes to 3.7021×10^6 and the difference comes to 3.6979×10^6. Students should sense-check their answers by noting in advance that both answers must be 'around 4 million'.

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