[MATHS PROBLEM]

ADDING FRACTIONS

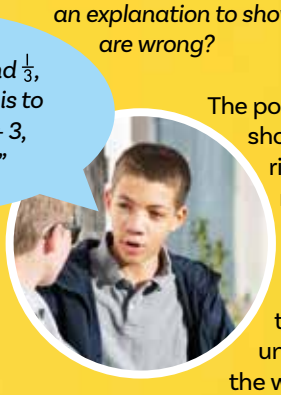
Students will often try to add fractions by separately adding the numerators and the denominators – this lesson by **Colin Foster** helps them to see why that can't be right...

Every student knows what the 'addition' symbol means. But the familiarity of this operation can lead them into misapplying it when dealing with fractions. This lesson helps students see that simply adding numerators and adding denominators *cannot* give the right answer, and leads them to see why the correct process is needed.

THE DIFFICULTY

This task places an error in the mouth of a fictional student, and gives the real students the job of correcting them.

What do you think about this? What would you say to this student?



Some students may agree with the fictional student, but most will probably realise from how the question is posed that something must be wrong. Certainly, the fictional student's method is 'easy' – but the real students will quickly realise that it can't be right.

Can you draw a picture to explain why the student is wrong? Can you come up with an explanation to show them that they are wrong?

The point here is to just show why $\frac{3}{8}$ can't be right, not necessarily to find the correct answer yet. It's important to first spend time understanding why the wrong way is wrong, before rushing to the correct method.

THE SOLUTION

Here are two ways of seeing why $\frac{3}{8}$ can't be right:

1. Think about size

Roughly how big are $\frac{2}{5}$, $\frac{1}{3}$ and $\frac{3}{8}$?

One way to answer this might be to convert them to decimals or percentages, perhaps using a calculator: $\frac{2}{5}$ = 0.4, $\frac{1}{3}$ = 0.333... and $\frac{3}{8}$ = 0.375. We can now see that $\frac{2}{5}$ by itself is more than $\frac{3}{8}$, so $\frac{2}{5} + \frac{1}{3}$ must be even more!

Another way to illustrate this would be to make sketches, using circles or bars of equal total size *(fig 1)*.

2. Think about equivalence

Write down some fractions that are equal to $\frac{2}{5}$.

Students might list $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30}$, etc.

Write down some fractions that are equal to $\frac{1}{3}$.

Students might list $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$, etc.

You can then highlight from these the two red fractions, which we certainly can add up, because their denominators are equal: *(fig 2)*

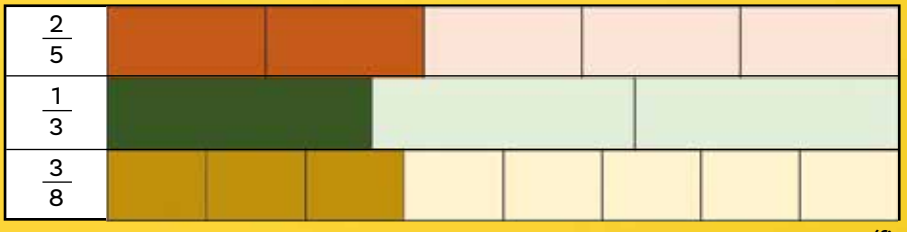
$$\frac{\cancel{2}^{\,6}}{\cancel{5}_{\,15}} + \frac{\cancel{1}^{\,5}}{\cancel{3}_{\,15}} = \frac{11}{15}$$

(fig 2)

This gives the correct answer, $\frac{11}{15}$, which is a much more sensible size!

Checking for understanding

This task will help to assess how students' understanding has developed – *Work out $\frac{1}{4} + \frac{2}{3}$ and explain in sentences how you know that your answer must be*



(fig 1)

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