Lesson plan: MATHS KS3

ALMOST ZERO

Sequences of decimals provide an opportunity for students to investigate patterns, says **Colin Foster**

Download a FREE set of KS3 place value problem cards for decimals at **teachwire.net/placevalue**

WHY TEACH THIS?

Decimals are all around us, and yet to many people are a bit of a mystery. This lesson aims to help students become more confident using them.

KEY CURRICULUM LINKS

+ Extend understanding of the number system and place value to include decimals

+ Understand and use place value for decimals

In this lesson, students explore decreasing arithmetic sequences in which the common difference is a decimal number. The task is to investigate whether the sequence includes zero, and if not, which term of the sequence is closest to zero. There are opportunities for students to generalise their findings and much routine practice of subtracting decimals is embedded in the task.

Q How close to zero do arithmetic sequences of decimal numbers get?

STARTER ACTIVITY

Q *Can you find three decimals which add up to 1?*

Q *Now find another three decimals which add up to 1 and*

- where two are the same size as each other and the third one is larger
- where two are the same size as each other and the third one is smaller
- where one is twice as large as another
- where one is smaller than 0.1
- where one is larger than 0.9
- where one is smaller than 0.1 and another is larger than 0.9

These various constraints are intended to help students to become fluent handling decimal numbers – confident, accurate, fast and flexible. Students may be able to make up their own similar decimal challenges for each other.

MAIN ACTIVITY

Q Let's start at 2.5 and count back in 0.3s: 2.5, 2.2, 1.9, 1.6, 1.3, 1.0, 0.7, 0.4, 0.1, -0.2, -0.5, -0.8, -1.1, -1.4, -1.7, -2.0, -2.3, -2.6, ...

Keep going long enough to pass through zero and into negative numbers.

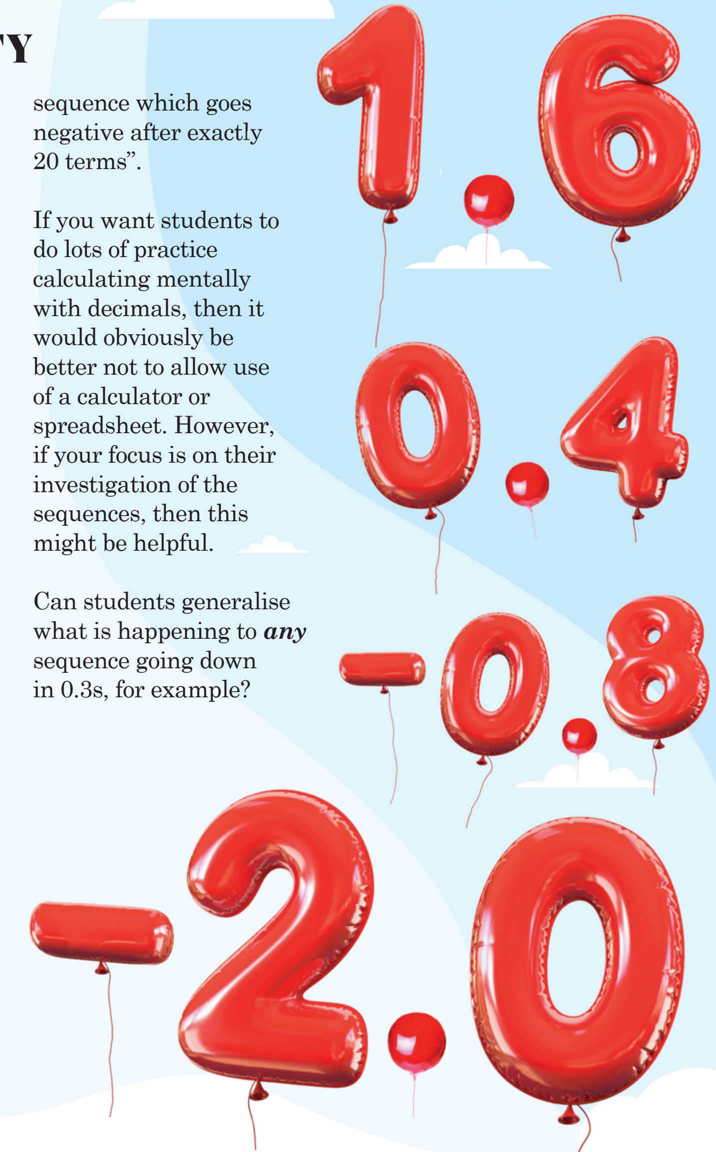
Q What mathematical questions could you ask about this sequence?

Students may pose questions about whether such a sequence hits zero, or some other number, such as -2.5 or -100, or which term in the sequence is closest to zero. They might also ask questions about how many terms it will take before the sequence 'goes negative'.

Encourage students to explore these sorts of questions, relating to this sequence and to other similar sequences. They can start by posing a question, making a prediction, and then testing it out by writing down the terms of the sequence. You might set a challenge such as "Find a sequence which goes negative after exactly 20 terms".

If you want students to do lots of practice calculating mentally with decimals, then it would obviously be better not to allow use of a calculator or spreadsheet. However, if your focus is on their investigation of the sequences, then this might be helpful.

Can students generalise what is happening to ***any*** sequence going down in 0.3s, for example?



DISCUSSION

Discuss the sequences that students have created, what they have found out and how they went about it.

Q. *Which questions did you pose? How did you try to answer them? What was difficult? What did you find out about how these sequences work?*

Students will not formulate their generalisations in a detailed symbolic fashion, but the following summary may be useful for the teacher to compare with.

Starting at the number a and going down in ds means that the sequence goes

$$a, a - d, a - 2d, a - 3d, \ldots,$$

and the nth term will be $a - (n - 1)d$, where $d > 0$.

If $\frac{a}{d}$ is an integer, then the sequence will hit zero exactly on the $(1 + \frac{a}{d})$th term, and therefore it will go negative on the $(2 + \frac{a}{d})$th term. Otherwise, the sequence will go negative on the $\lceil 1 + \frac{a}{d} \rceil$th term, where the brackets indicate ***rounding up*** to the next integer.

If $2a + (1 - 2\lceil \frac{a}{d} \rceil)d < 0$, then the term nearest zero will be the positive term $a - (\lceil \frac{a}{d} \rceil - 1)d$.
If $2a + (1 - 2\lceil \frac{a}{d} \rceil)d > 0$, the term nearest zero will be the next term, $a - \lceil \frac{a}{d} \rceil d$, which will be negative.
Finally, if $2a + (1 - 2\lceil \frac{a}{d} \rceil)d = 0$, then there is no nearest term to zero, since the $\lceil \frac{a}{d} \rceil$th term and the $\lceil 1 + \frac{a}{d} \rceil$th term are equidistant from zero.

ADDITIONAL RESOURCES

A related task is available at **nrich.maths.org/10326**

ONLINE EXTRAS

To explore 10 of the best fraction resources for secondary maths – whether it's adding and subtracting, arranging in size order or finding equivalents – visit **tinyurl.com/tsfractions**

GOING DEEPER

Confident students could explore what happens with more complicated sequences, such as where the difference doubles every time; for example, subtracting 0.3 to get the second term, but then subtracting 0.6 to get the third term and 1.2 to get the fourth term, etc.

HOME LEARNING

For a Don Steward homework task involving operations on decimals, see **tinyurl.com/tsdonsteward** – a SMILE resource using decimals is available at **tinyurl.com/tssmiledecimals**

THE AUTHOR

Colin Foster is an Associate Professor in the School of Education at the University of Leicester. He has written many books and articles for mathematics teachers (see **www.foster77.co.uk** and **@colinfoster77** on Twitter).