ANGLE CHASING

THE ANGLES CREATED BY OVERLAPPING REGULAR POLYGONS CAN BE EASY OR DIFFICULT TO CALCULATE, DEPENDING ON WHAT INFORMATION YOU ARE GIVEN, SAYS COLIN FOSTER...

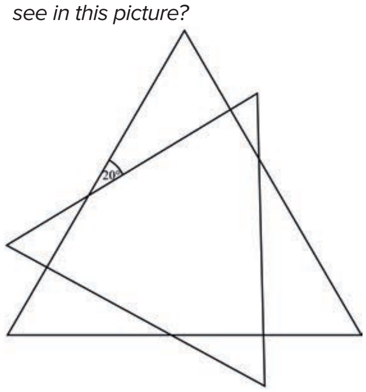
WHY TEACH THIS?

Taking a creative approach to important mathematical processes – such as calculating angles within geometrical shapes – can deepen students' understanding of the principles involved and develop vital skills of reasoning and deduction. Solving these problems will increase students' confidence both in this specific topic and in their power to deal with complex situations more generally.

Many routine exercises involving calculating angles are one-step problems where, for example, two angles in a triangle are given and the third is to be found. These are unlikely to lead to higher-level thinking and reasoning or to stimulate much excitement about mathematics. 'Angle chasing' refers to problems in which longer chains of reasoning are necessary, where students need to make choices and be strategic in order to find the required angles. The problems in this lesson arise from overlapping polygons and depend only on knowledge of simple angle facts, such as the angle sum at a point, on a straight line and inside a triangle, but they are tricky because deciding which intermediate angles will help in finding the required one involves considerable thought. Students can create their own angle-chasing problems by combining different polygons in various positions and they can explore alternative solutions. Some of these problems can be straightforward while others are quite demanding, allowing students to tackle problems that offer them an appropriate level of challenge.

STARTER ACTIVITY

Q. Can you describe what you see in this picture?



This diagram, and all the others in this lesson, are available at www.foster77.co.uk/Angle%20Chasing%20TASK%20SHEET.pdf

Students might comment that they see two overlapping triangles or they might focus on some of the six smaller triangles produced. They might see an irregular hexagon in the middle or other shapes. They might conjecture that the two large triangles are equilateral. They might comment on the 20° angle and whether they think that it is drawn accurately or not.

Q. Let's suppose this picture is formed by taking two equilateral triangles and overlapping them. Which angles in the picture do you think you would be able to work out the size of?

Students might need to remind each other what an 'equilateral triangle' is. (All three sides have the same length and all of the angles are 60°.) The easiest angles to find are the six 60° angles in the equilateral triangles and the 300° 'outside' angles at the same points. To find the other missing angles, students will need to use the facts that vertically opposite angles are equal, that angles on a straight line add up to 180° and that the sum of the angles in a triangle is 180°.

Q. I would like you to try to find as many of the angles in this diagram as you can. Make sure you have a good reason for each answer.

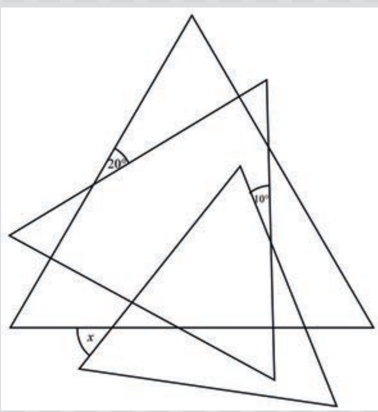
Students could begin doing this together as a whole class, and after you have found a few answers together they could work on this task in pairs. You could give them a photocopy of the drawing or ask them to make their own sketches. Some pairs might wish to make an accurate drawing of the diagram, as opposed to a mathematical sketch. They could use ruler and compasses to construct the equilateral triangles (their exact sizes don't matter) or measure the sides and use a protractor to get the 60° angles. Either way, students will need a protractor to draw the 20° angle.

Students should find six similar 20°-60°-100° triangles, six 80° angles and six 160° angles. So there are 36 possible angles altogether but only 6 different sizes of angle. Encourage students to explain how they know the sizes of the angles that they have found.

If students have found this very hard, they could perhaps try the same thing again with 30° instead of 20° as the given angle. Those who have found it easy could generalise to an '*a*' angle where the 20° angle was.

MAIN ACTIVITIES

Q. Now we are going to make it a bit more complicated by adding in a third equilateral triangle.



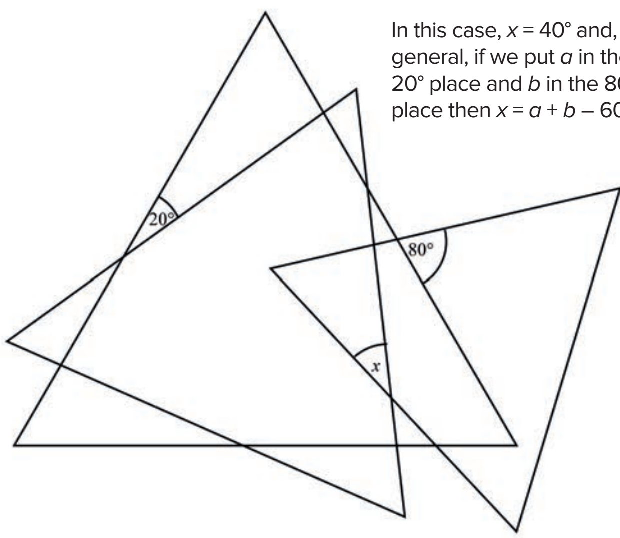
Q. Can you find all the angles this time? In particular, can you find angle x? I would like you to work in pairs.

It will probably help if each pair has their own copy of this diagram – if possible, enlarged onto A3 paper, so that it is big enough for the students to annotate it and so that both students in the pair can see what is going on. Knowing that the angles inside a quadrilateral add up to 360° could be useful here but is not essential.

Students might like to estimate the size of x before they begin. Some pairs might like to make accurate drawings to confirm the results of their reasoning.

The answer is $x = 50°$. A possible extension is to see how x depends on the 20° and 10° given in the diagram by changing one or both to some other value and finding x again. In general, if we have a in the 20° place and b in the 10° place then $x = 60 + b – a$.

It is easy for students to invent other positions for the third equilateral triangle and 'x' so as to generate further puzzles. One example is shown below, and this and another are given at **www.foster77.co.uk/Angle%20Chasing%20TASK%20SHEET.pdf.**



In this case, $x = 40°$ and, in general, if we put a in the 20° place and b in the 80° place then $x = a + b – 60$.

SUMMARY

You could conclude the lesson with a plenary in which the students talk about what they have found out and learned. Did any pair have a quick way of finding one of the answers? Which solutions do you find easier to understand? Why? Which solutions are more efficient by involving fewer steps? Did anyone find short-cuts, such as imagining rotating one triangle into another, which can sometimes enable students to 'see' the answer without doing as many calculations?

INFORMATION CORNER

ABOUT OUR EXPERT

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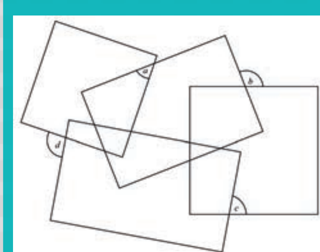
STRETCH THEM FURTHER

STUDENTS COULD INVENT PROBLEMS INVOLVING SQUARES AND RECTANGLES AS WELL AS, OR INSTEAD OF, EQUILATERAL TRIANGLES. HERE, A KNOWLEDGE OF CORRESPONDING AND ALTERNATE ANGLES ON PARALLEL LINES IS USEFUL. FOR EXAMPLE, ONE PROBLEM WOULD BE TO FIND AN ALGEBRAIC CONNECTION BETWEEN ANGLES *A, B, C* AND *D* IN THE DIAGRAM BELOW.



ADDITIONAL RESOURCES

DIAGRAMS FOR THESE PROBLEMS ARE ALL AVAILABLE AT **www.foster77.co.uk/Angle%20Chasing%20TASK%20SHEET.pdf** IF STUDENTS KNOW HOW TO USE GEOGEBRA **www.geogebra.org/cms/en/** THEN THAT COULD PROVIDE A VERY USEFUL ENVIRONMENT IN WHICH TO WORK ON THESE TASKS.