[MATHS PROBLEM]

AREA AND PERIMETER

Students often confuse area and perimeter, giving the answer to one when asked for the other. This lesson, by **Colin Foster**, aims to help students resolve this confusion...

Area and perimeter are really quite different quantities, but they are often cued by similar prompts, such as a drawing of a rectangle, with measurements written along two adjacent sides. In this lesson, students will explicitly contrast area and perimeter to see what is the same and what is different about them.

THE DIFFICULTY

This task is intended to bring to the surface students' confusions between area and perimeter.

*Draw me two rectangles: one with an **area** of 24cm² and one with a **perimeter** of 24cm. Clearly label which one is which.*

Students could do this task on mini-whiteboards or in their books. It would be easier to do on a squared background. Students are likely to produce a mixture of correct and incorrect responses, including confusions between area and perimeter.

One informal way to illustrate the difference is to think of ***perimeter*** as the amount of ink needed to ***draw*** the shape, and ***area*** as the amount of ink needed to ***colour it in***. It might be helpful to refer to the boundary of the school field as the ***perimeter*** of the field.

Find as many examples as you can of rectangles with an area of 24cm² and rectangles with a perimeter of 24cm.

A 24cm² ***area*** rectangle with integer sides must be 1 × 24, 2 × 12, 3 × 8 or 4 × 6. A 24cm ***perimeter*** rectangle with integer sides must be 1 × 11, 2 × 10, 3 × 9, 4 × 8, 5 × 7 or 6 × 6 (the last one is square, but a square is just a special rectangle). But students may also find examples in which the sides are not integers.

THE SOLUTION

The tasks here are designed to focus students' attention on the value of the perimeter and the value of the area for the same rectangle, forcing them to constantly switch between the two as they compare and contrast them.

1. A rectangle with area = perimeter = 24

*Can a single rectangle have **both** an area of 24cm² **and** a perimeter of 24cm?*

Students might think not, because they only consider rectangles with integer sides. If so, you could ask them which integer rectangle comes ***closest*** to having area and perimeter both 24. The closest ones are the 3 × 9 rectangle, with a perimeter of 24cm but an area of 27cm², and the 3 × 8 rectangle, with an area of 24cm² but a perimeter of 22cm. However, allowing ***non-integer*** sides could enable students to experiment on calculators to find that a 2.536... × 9.464... rectangle has area and perimeter both equal to 24. (The exact dimensions are $6 + 2\sqrt{3}$ and $6 - 2\sqrt{3}$.)

2. Equable rectangles

Find some other rectangles that have ***equal*** area and perimeter.

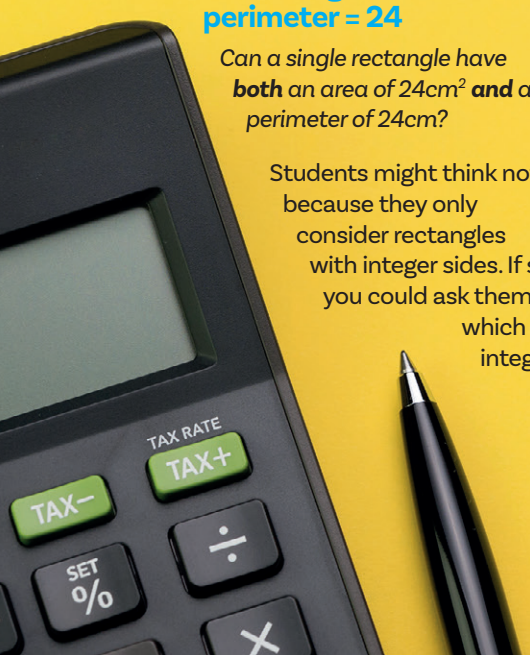
Students may think that a 2 × 2 square satisfies these conditions, because 2 + 2 = 2 × 2, but 2 + 2 is only the ***semi***-perimeter, so it is actually a 4 × 4 square that satisfies the conditions. The only other equable rectangle with integer sides is a 3 × 6, but there are infinitely many with non-integer sides; in general an $l \times \frac{2l}{l-2}$ rectangle has area = perimeter = $\frac{2l^2}{l-2}$.

Students confident with Pythagoras's Theorem could attempt to find equable ***triangles***. It turns out that there are only 5 different equable integer triangles; the two which are right-angled (6-8-10 and 5-12-13) are easier to find than the other three (6-25-29, 7-15-20 and 9-10-17) (see Bradley, 2005, pp. 15-16).

Checking for understanding

*Draw me two rectangles: one with an area **greater than** perimeter and one with an area **less than** perimeter. Clearly label which one is which and work out the area and the perimeter of both of them.*

Bradley, C. J. (2005). Challenges in geometry for mathematical Olympians past and present. OUP.



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