[MATHS PROBLEM]

AREA AND VOLUME SCALE FACTORS

When comparing similar shapes, students are often confused about linear, area and volume scale factors

In this lesson, students explore what happens to the area of a disc when the disc is enlarged.

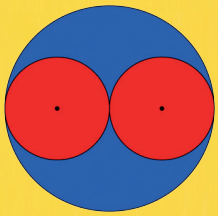
THE DIFFICULTY

Imagine that you ring up and order a 20-inch pizza to be delivered. When they arrive, they say they are sorry, they don't have any 20-inch pizzas, but they have brought you two 10-inch pizzas instead. Is that OK?

Students may worry about things like the amount of crust, the shape of the slices, or how the pizzas would be shared out, but not realise that in fact two 10-inch pizzas contain only ***half as much*** pizza as one 20-inch pizza!

THE SOLUTION

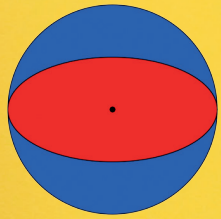
Rather than explaining the point, just display the image below, without any words, and ask: *What does this have to do with the pizza question?*



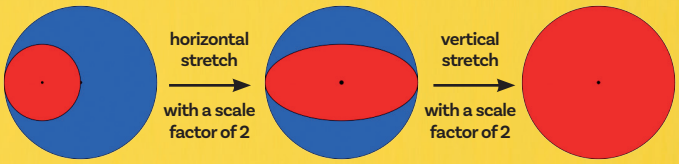
If the diameter of the blue disc is 20 inches, then the diameters of the red discs will be 10 inches. The total area of the two red discs is clearly much less than the area of the blue disc.

Since the area of a disc is πr^2, where r is the radius, Area $\propto r^2$, or, since radius $\propto$ diameter, d, Area $\propto d^2$. This means that ***twice as big*** a diameter gives a disc with ***four times*** as much area. The two red discs in total cover only ***half*** of the area of the blue disc!

Another way to think about it is to imagine ***stretching*** one of the red discs until it is twice as wide, but without changing its height.



It now becomes an ellipse, with ***twice*** the area of the original red disc. This means that the area of the ellipse is equal to the sum of the areas of the two red discs. This red area is half of the area of the blue disc, because the red ellipse would have to be stretched ***vertically*** by another scale factor of 2 for it to become congruent to the blue disc. This would take ***another*** doubling of its area to match the area of the blue disc:



Students may think this is just a weird thing about circles, but it is actually completely general. Whenever ***any*** 2D shape is enlarged, for the enlarged version to remain similar to the original shape, it must be stretched by the same factor k both horizontally and vertically. Each of these stretches increases the area by a factor of k, so overall the area will increase by a factor of k^2.

Checking for understanding

To assess students' understanding, ask them to imagine scaling up a sphere in 3D. *What would happen to the volume?*

Multiplying the radius (or diameter) of a sphere by a factor of k will require ***three*** stretches this time, in order for the new shape to be spherical (i.e., similar to the original sphere). The three stretches, each with scale factor k, in all three perpendicular directions, will scale up the volume by a factor of k^3.

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