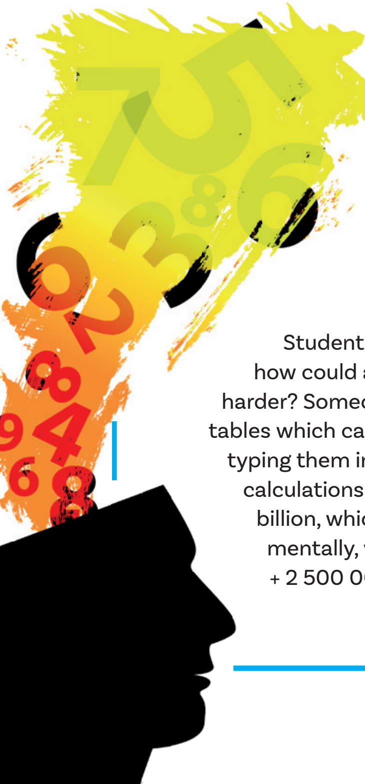
Lesson plan: MATHS KS4

BEAT THE CALCULATOR!

Sometimes a technological approach is neither the best nor the easiest way to tackle calculations, says **Colin Foster**

In this lesson, students have to create calculations which can be done more quickly or accurately *without* using a calculator. This helps to encourage them to be more critical and careful about their use of calculators. It also provides some opportunities for students to make use of their skills in simplifying algebraic expressions.



STARTER ACTIVITY

Q *Can you think of a calculation which is easier to do* ***without*** *using a calculator?*

Students might be a bit puzzled by this – how could a calculator ever make something harder? Someone might suggest 1 + 1 or times tables which can be retrieved much faster than typing them into a calculator. There are also calculations such as 2.5 billion + 2.5 billion = 5 billion, which can be answered very quickly mentally, whereas typing in '2 500 000 000 + 2 500 000 000 =' would take much longer.

Download a fantastic KS4 lesson plan on patterns in calculations at **teachwire.net/calpatterns**

WHY TEACH THIS?

Calculators are powerful tools, but they need to be used intelligently and critically, checking that the answers that they give make sense.

KEY CURRICULUM LINKS

+ Consolidate their numerical and mathematical capability from key stage 3 and extend their understanding of the number system to include powers, roots and fractional indices

+ simplify and manipulate algebraic expressions

KEY QUESTION

Are there calculations which can be done more quickly or accurately without a calculator?

MAIN ACTIVITIES

Q Are there any calculations which you can do more *accurately* than the calculator can?

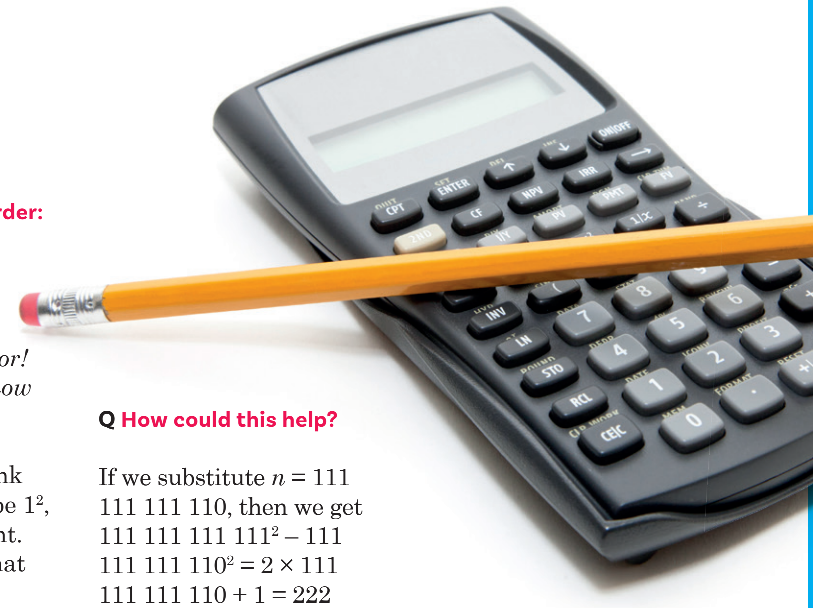
Students may not have any ideas for this one. If they suggest anything, they could try it. If not, you could suggest:

111 111 111 111 + 1

Before trying it on a calculator, can they see what might go wrong?

Most calculators will not have enough memory capacity or display space to give all the digits in the answer 111 111 111 112, and so will give an output in standard form as something like 1.111 11 × 10^{11}. This answer is only approximately correct, although the calculator will not draw our attention to this, which is something to be aware of. Students may be surprised that if they enter 1 × 10^{11} + 1 the calculator gives an answer of 1 × 10^{11} – the "add 1" makes no difference to the answer displayed!

Q Are there any calculations which you can do more accurately than the calculator can?



Q Here is something harder:

$111\,111\,111\,111^2$
$-\ 111\,111\,111\,110^2$

Don't trust the calculator! Do you have any idea how to work it out?

Some students will think that the answer must be 1^2, or 1, but this is not right. You could ask them what $11^2 - 10^2$ is equal to.

They should be able to work this out in their heads – and a calculator will have no trouble with this one – and they will see that the answer is not 1. They could explore patterns in $10^2 - 9^2$, $9^2 - 8^2$, etc. to try to determine what is going on.

Q Could we use algebra to help?

The idea here would be to capture the structure of what is happening when you find the difference between the squares of two consecutive numbers:

$(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$

Q How could this help?

If we substitute n = 111 111 111 110, then we get $111\,111\,111\,111^2 - 111\,111\,111\,110^2 = 2 \times 111\,111\,111\,110 + 1 = 222\,222\,222\,221$, so that must be the exact answer. The calculator probably won't tell us about the 1 on the end.

Q What other 'hard calculations' could we do using this algebraic identity?

For example, we could do

$444\,444\,444\,444\,444\,443^2 - 444\,444\,444\,444\,442^2 = 888\,888\,888\,888\,888\,885$

Q Here are some more to try. What other ones can you invent?

$111\,111\,111\,113^2$
$-\ 111\,111\,111\,111^2$

$$\frac{444\,444\,444\,444^2}{888\,888\,888\,888}$$

$$\frac{222\,222\,222\,222^2}{111\,111\,111\,111^2}$$

$$\frac{222\,222\,222\,222^2 - 111\,111\,111\,111^2}{222\,222\,222\,222 - 111\,111\,111\,111}$$

Calculators will at least give a rough sense of the size of the answer, so may be useful to some extent.

One way for students to create their own examples is to start by making up some algebra that simplifies nicely, and then choosing the numbers to go in.

DISCUSSION

You could conclude the lesson by discussing how students worked out these calculations. It is their job to convince everyone else that their answers are correct.

Q. *How did you tackle these? Who used algebra? Who had other ways of doing them?*

Different students may have used algebra differently, depending on which number in the calculation they decided to replace by a letter, and it would be useful to discuss this.

The first one could be approached by using

$(n + 2)^2 - n^2 = n^2 + 4n + 4 - n^2 = 4n + 4$

with n = 111 111 111 111, giving

$111\,111\,111\,113^2 - 111\,111\,111\,111^2 = 444\,444\,444\,448$.

For the second one, it is natural to let n = 444 444 444 444, giving, $\frac{n2}{2n} = \frac{n}{2}$, so $\frac{444\,444\,444\,444^2}{888\,888\,888\,888}$ = 222 222 222 222. Algebra might not really be needed here, as students might see

$\frac{444\,444\,444\,444 \times 444\,444\,444\,444}{2 \times 444\,444\,444\,444}$ and then cancel down.

Similar approaches work for the third one, giving $\frac{222\,222\,222\,222^2}{111\,111\,111\,111^2} = 4$.

The last one could be thought of as $\frac{(2n)^2 - n^2}{2n - n} = 3n$, giving $\frac{222\,222\,222\,222^2 - 111\,111\,111\,111^2}{222\,222\,222\,222 - 111\,111\,111\,111}$ = 333 333 333 333.

Give students time to share the ones they created, and explain how they came up with them, and then invite other students to work them out.

ADDITIONAL RESOURCES

wolframalpha.com allows calculations to a huge number of significant figures, and so could be used to check some of the trickier calculations. Modern technology allows very accurate calculations, but still needs to be used critically.

GOING DEEPER

Confident students could construct calculations with increasingly complicated structures, in which lots of things cancel out, using whatever properties they know. For example, raising a complicated expression to the power of zero will reduce it to 1.

THE AUTHOR

Colin Foster is an Associate Professor in the School of Education at the University of Leicester. He has written many books and articles for mathematics teachers (see **www.foster77.co.uk** and **@colinfoster77** on Twitter).