[MATHS PROBLEM]

COMPLETING THE SQUARE

Many students find completing the square a difficult process to get to grips with

In this lesson, students make sense of completing the square by ordering the steps and explaining the process in pairs.

THE DIFFICULTY

Give students the 13 statements below for them to cut out onto separate strips (a resource sheet for this is available at XXX).

Arrange some of these strips in order to show the 'completing the square' method. You won't need to use all of the strips!

If students find this too hard, you could remove some of the unnecessary strips. The red numbers indicate a possible order, although some of these might be omitted. The other ones are not needed at all.

You could ask students who complete this easily to design questions for which some of the unused strips would be needed.

$x = 5 \pm 2$	
$(x + 5)^2 + 4 = 0$	
$(x + 5)^2 - 25 + 21 = 0$	2
$x + 5 = \pm 2$	5
$(x + 5 - 2)(x + 5 + 2) = 0$	
$x^2 + 10x + 21 = 0$	1
$x = -7$ or -3	7
$(x + 5)^2 = \pm 4$	
$(x + 5)^2 = 4$	4
$x = -5 \pm 2$	6
$(x + 5)^2 - 4 = 0$	3
$(x + 3)(x + 7) = 0$	
$x + 5 = \pm 4$	

THE SOLUTION

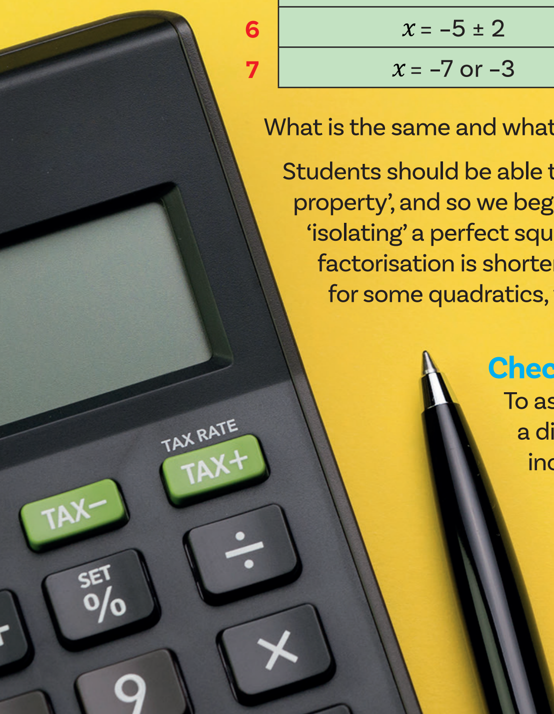
Now find and order strips within this same set that demonstrate the 'factorisation' method.

There is a shorter and a longer way of showing factorisation:

	completing the square	*factorisation*	
		shorter version	*longer version*
1	$x^2 + 10x + 21 = 0$	$x^2 + 10x + 21 = 0$	$x^2 + 10x + 21 = 0$
2	$(x + 5)^2 - 25 + 21 = 0$	$(x + 3)(x + 7) = 0$	$(x + 5)^2 - 25 + 21 = 0$
3	$(x + 5)^2 - 4 = 0$	$x = -7$ or -3	$(x + 5)^2 - 4 = 0$
4	$(x + 5)^2 = 4$		$(x + 5 - 2)(x + 5 + 2) = 0$
5	$x + 5 = \pm 2$		$(x + 3)(x + 7) = 0$
6	$x = -5 \pm 2$		$x = -7$ or -3
7	$x = -7$ or -3		

What is the same and what is different about completing the square and factorisation?

Students should be able to identify many differences. For example, factorisation depends on the 'zero product property', and so we begin by getting all the terms on the same side. Completing the square depends on 'isolating' a perfect square ($(x + 5)^2$ above) and then square-rooting both sides. Students might think factorisation is shorter or easier than completing the square; they might note that factorisation works only for some quadratics, whereas completing the square works for all.



Checking for understanding

To assess students' understanding, ask them to create their own set of strips like this, with a different starting equation – for example, $x^2 = 3x + 18$ – and ask them to make sure to include some 'distractor' strips that don't belong!

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