CONNECTED QUADRATICS

When learners are bored in maths lessons they sometimes ask, "When will we ever actually use this?" But even topics that are unlikely to feature in everyday life can still be fun for learners to work on if they experience them as a puzzle to solve, says **Colin Foster...**

TODAY YOU WILL...

PRACTISE SOLVING QUADRATIC EQUATIONS BY FACTORISING WHILE WORKING ON A DEEPER PROBLEM

How can learners become confident and fluent with techniques such as factorising quadratics without simply ploughing through pages of exercises? One way is to offer a scenario in which they need to construct their own quadratic equations in order to solve a bigger problem. Lots of practice happens along the way, while something a bit more interesting is going on.

STARTER ACTIVITY

Q: *Do you remember how to solve a quadratic equation like:*
$x^2 - 7x - 30 = 0$?

The easiest way is to factorise the left-hand side:

$(x + 3)(x - 10) = 0$

(The 3 and the –10 come from the fact that their sum is –7, the coefficient of x, and their product is –30, the constant term.)

Now if two numbers multiply to make zero, either the first one is zero or the second one is zero. So in this case, either $x + 3 = 0$ or $x - 10 = 0$, giving the two answers $x = -3$ or 10.

Q: *What if I change the starting equation by sticking a '2' on the front:*
$2x^2 - 7x - 30 = 0$?

You can still solve it by factorising, but the factorising is harder now because the left-hand side is *non-monic* (i.e., the coefficient of x^2 is not 1). Sometimes learners do this by trial and error, but a more systematic method is to place x's in two brackets and write the coefficient of x (which is 2, here) three times:

$$\frac{(2x\quad)(2x\quad)}{2}$$

Then put in two numbers that have a sum of –7 (the coefficient of x, as before), but this time they need to have a product of $2 \times (-30) = -60$, the product of the coefficient of x^2 and the constant term. The two numbers required here are 5 and –12, so you get:

$$\frac{(2x+5)(2x-12)}{2}$$

which simplifies to $(2x + 5)(x - 6)$.

There are other perfectly good ways of factorising non-monic quadratics, but this is a method learners (or even teachers) may not have seen [Note 1, below].

So $(2x + 5)(x - 6) = 0$, meaning that either $2x + 5 = 0$ or $x - 6 = 0$, giving $x = -2.5$ or 6.

Learners could try solving $x^2 + 17x + 30 = 0$ and $2x^2 + 17x + 30 = 0$ to see if they can do them themselves. (The answers are $x = -15$ or –2 and $x = -2.5$ or 3, respectively.)

MAIN ACTIVITIES

1 CONNECTED QUADRATICS

Q: *Along each side of the triangle, make the two quadratic expressions equal and solve the equation.*

$5x^2 + 8x + 2$

$3x^2 + 5x + 1$ —— $2x^2 + 9x + 6$
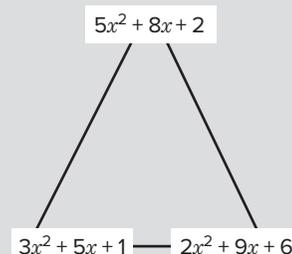
1 This method comes from Lyszkowski, R. M. (1999) A simple method for factorizing expressions of the form ax²+ bx+ c, Mathematics in School, 28, 34.

This triangle of expressions leads to three equations, all of which can be solved by factorising. The equation formed along the bottom side of the triangle is monic, so learners could start with that one, as it is the easiest; the other two are non-monic. Learners could write their solutions along the sides of the triangle:

$$5x^2 + 8x + 2$$

–1 or –0.5   4/3 or –1

$$3x^2 + 5x + 1 \quad 2x^2 + 9x + 6$$

–1 or 5
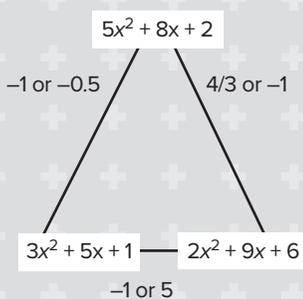
2 MAKE UP YOUR OWN

Q: *See if you can make up your own triangle of connected quadratics. Remember, all three equations must be solvable by factorising!*

This is quite hard and will engage learners in a lot of useful trial and improvement practice at solving quadratics (or trying to!) They will realise the important fact that not all quadratics have real solutions, and even those that do cannot necessarily be solved by factorising.

Additional (harder) challenges could be:

Q: *Can you make all the solutions integers?*

Q: *Can you make a square of connected quadratics, forming four equations, each of which can be solved by factorising?*

Q: *Can you make a square of connected quadratics, including the diagonals, so that there are six equations, each of which can be solved by factorising?*

INFO BAR

+ STRETCH THEM FURTHER

+ THE LESSON BEGAN WITH THE FACTORISABLE EXPRESSION $x^2 - 7x - 30$, WHICH WAS ALSO FACTORISABLE WHEN A '2' WAS ADDED ON THE FRONT TO GIVE $2x^2 - 7x - 30$. LEARNERS MIGHT LIKE TO TRY TO FIND OTHER MONIC QUADRATICS WHICH ARE FACTORISABLE BOTH BEFORE AND AFTER STICKING A '2' (OR PERHAPS SOME OTHER INTEGER) ON THE FRONT.

YOU CAN FIND OUT MORE ABOUT THIS AT FOSTER, C. (2012) QUADRATIC DOUBLETS, MATHEMATICAL GAZETTE, 96(30), 264–266.

+ ADDITIONAL RESOURCES

+ A REALLY NICE RESOURCE, PARTICULARLY FOR USE ON AN ELECTRONIC WHITEBOARD, IS TINYURL.COM/CBYW4YX THERE ARE SOME SUGGESTIONS ABOUT WAYS OF USING IT AT TINYURL.COM/CZUUGLU, BUT THE BASIC IDEA IS FOR LEARNERS TO CONJECTURE ABOUT THE CONTENTS OF THE CELLS IN THE GRID, BASED ON THEIR AWARENESS OF THE STRUCTURE, AS OTHER CELLS ARE REVEALED. EACH CELL CONTAINS A QUADRATIC EXPRESSION IN BOTH FACTORISED AND EXPANDED FORM.

+ ABOUT THE EXPERT

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(With additional thanks to Dougald Tidswell)

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3 POSTER EXPLANATIONS

Learners might like to design a clear and attractive poster to illustrate how to solve quadratic equations by factorising. They could tackle monic or non-monic cases, or both.

HOME LEARNING

Learners could try to construct a triangle in which one of the expressions is quadratic and the other two are linear (i.e., so-many lots of x, plus or minus a number; no x^2 term). This should lead to two quadratic equations and one linear equation, all of which should give integer answers.

SUMMARY

You could conclude the lesson with a plenary in which learners share their creations and show the different expressions that they have constructed. How did they go about it? What problems did they encounter? Could they explain to someone else how to do it?