[MATHS PROBLEM]

CONVERTING COMPOUND UNITS

Students often get confused when working with different compound units – such as converting miles per hour to metres per second, says **Colin Foster**

In this lesson, students think systematically about how to convert compound units.

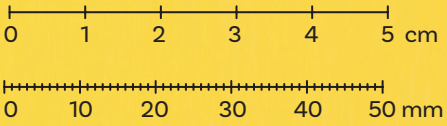
THE DIFFICULTY

Which is faster: 30mph or 30m/s?

The answer isn't obvious. Students might say that metres are smaller than miles, so 30m is much less than 30 miles. But, on the other hand, hours are longer than seconds. The issue is whether hours are ***sufficiently*** longer than seconds to compensate for the extent to which metres are shorter than miles. Without calculation, it's impossible to be sure.

THE SOLUTION

Let's begin with something easier.
Look at a ruler that has both cm and mm scales.



Which of these is true?

A cm $\xrightarrow{\div 10}$ mm

or

B cm $\xrightarrow{\times 10}$ mm

Depending on what is meant, either of these could be true:

- A indicates that if you take a cm and divide it into 10 equal lengths, you get a mm.
- B indicates that the ***number of*** cm multiplied by 10 gives the ***number of*** mm; for example, 5cm = 50mm.

Students can be confused about which is needed. To find out how many 'new' units are equivalent to a certain number of 'old' units, we need (B), not (A).

Let's convert 30mph into m/s:

mph $\xrightarrow{\times 1.6}$ km/h $\xrightarrow{\times 1000}$ m/h $\xrightarrow{\div 60}$ m/min $\xrightarrow{\div 60}$ m/s

If we work out $\frac{1.6 \times 1000}{60 \times 60}$ we get $\frac{1600}{3600} = \frac{4}{9}$. To convert a speed in mph to a speed in m/s you just have to multiply by $\frac{4}{9}$. To go the opposite way we have to ***divide*** by $\frac{4}{9}$; alternatively, we can ***multiply*** by $\frac{9}{4}$. So, 30mph = 13.3m/s and 30m/s = 67.5mph.

If we only need an approximate answer, we could approximate $\frac{4}{9}$ as $\frac{1}{2}$, so 30mph is about 15m/s and 30m/s is about 60 mph.

Another common compound quantity is density. The density of gold, for example, is about 19g/cm^3. To convert this into kg/m^3, we need to first divide by 1000, to convert g to kg, and then multiply by 1 000 000, to convert cm^3 into m^3.

Students may think that the second conversion factor should be 100, since there are 100cm in 1m. But if they imagine a 100cm x 100cm x 100cm cube, they'll see it contains 1 000 000cm^3 – *not* 100cm^3. Dividing by 1000, and then multiplying by 1 000 000 is equivalent to multiplying by 1000, which means the density of gold will be about 19 000kg/m^3.

Checking for understanding

In the UK, we measure fuel efficiency in 'miles per gallon'. Elsewhere, people use 'litres per 100 km'. Work out how to convert between these. Which is more – 30mpg or 30l/100 km?

These units are ***inversely*** related. The more fuel efficient a vehicle is, the ***greater*** its mpg will be, because it will travel ***more*** miles on each gallon of fuel. But its l/100 km will be ***less***, because it will use up ***fewer*** litres of fuel for every 100 km travelled.

We need to divide 280 by one of these quantities to obtain the other. A 30mpg vehicle, for example, has a fuel efficiency of $\frac{280}{30}$ l/100 km, or about 9l/100 km.

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