[MATHS PROBLEM]

DIVIDING FRACTIONS

Students are often confused about dividing by a number that is a fraction

In this lesson, students make sense of fraction division by drawing bar diagrams.

THE DIFFICULTY

This task is intended to bring to the surface students' confusions about division of fractions.

What is 6 divided by one half?

Many students will think that the answer is 3, whereas the correct answer is actually 12. You could respond to students who say '3' by asking another question:

What is 6 divided by 2?

They will be sure that the answer to that question is 3, and this may make them doubt ***that*** $6 \div \frac{1}{2}$ can really also be 3.

THE SOLUTION

Bar models can be a good way to address what is going on. We can draw a bar for 6:

and a bar for 2:

And it is clear that 3 of the red bars will fit exactly into the yellow bar.

This corresponds to $6 \div 2 = 3$. (How many 2s fit into 6? The answer is 3.)

What would a bar of length $\frac{1}{2}$ look like?

It will be.

and so it is clear that **12** of these purple bars will fit exactly into the yellow bar:

So, $6 \div \frac{1}{2} = 12$. (How many $\frac{1}{2}$s fit into 6? The answer is 12.)

Can you draw or imagine bars to work out $5 \div \frac{1}{2}$, $2 \div \frac{1}{3}$ and $4\frac{1}{2} \div \frac{1}{4}$?

The answers to all of these (10, 6, 18) are integers, but students also need to be able to divide fractions in cases where the answer is ***not*** an integer.

We have seen that $6 \div \frac{1}{2} = 12$.

If we started with a number that was $\frac{1}{7}$ as big, the answer would also be $\frac{1}{7}$ as big:

So, $\frac{6}{7} \div \frac{1}{2} = \frac{12}{7}$.

Now for the hard bit.

If the fraction we ***divided by*** were ***5 times as big***, the answer would be ***5 times smaller***.

So, $\frac{6}{7} \div \frac{5}{2} = \frac{12}{35}$.

This shows us that $\div \frac{5}{2}$ is equivalent to $\times \frac{2}{5}$, because $\frac{6}{7} \times \frac{2}{5} = \frac{12}{35}$.

So, to divide by a fraction, we can instead ***multiply by its reciprocal***. (This is analogous to how subtracting 8 is equivalent to ***adding*** the additive inverse, which is −8.)

Checking for understanding

These questions will help to assess how well students have understood this way of thinking.

*Explain why $5 \div \frac{1}{4}$ must be **larger** than 5.*

Explain why $5 \div \frac{1}{4}$ is equal to 20.

Use this to explain why $\frac{5}{3} \div \frac{1}{4}$ must be equal to $\frac{20}{3}$.

Use this to explain why $\frac{5}{3} \div \frac{11}{4}$ must be equal to $\frac{20}{33}$.

Students may benefit from some time in pairs to refine their explanations before sharing them.

They could then create their own fraction divisions to explain.

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