[MATHS PROBLEM]

EXPANDING PAIRS OF BRACKETS

Students are often confused about how to expand pairs of brackets in algebra, says **Colin Foster**

In this lesson, students connect expanding single brackets to expanding double brackets

THE DIFFICULTY

Which of the following is the odd one out and why?

$$3x + 6 \qquad 3(x + 6) \qquad 3x + 18$$

Students might give different answers. For example, $3(x + 6)$ could be the odd one out because it contains brackets. Alternatively, $3x + 18$ could be the odd one out because it contains a double-digit number (or doesn't contain a 6).

Why could $3x + 6$ be the odd one out?

It's because this expression isn't equal to the other two expressions (which are equal to each other).

$3(x + 6) = 3x + 18$ by **expanding** the brackets
$3x + 18 = 3(x + 6)$ by **factorising**.

THE SOLUTION

*How would you explain **why** $3(x + 6)$ **must** be equal to $3x + 18$?*

It's important to realise that this means the two expressions are equal for **every possible value** of x.

Suppose that $x = 10$. What will each of the three expressions be equal to?

Encourage students to see that $3 \times (10 + 6) = 3 \times 10 + 18$ ***without working out the numbers***; otherwise, it may seem like a fluke that 3×16 happens to be equal to $30 + 18$.

One way to see this is by 'stacking' the (10 + 6)s:

$$\begin{aligned} 3 \times (10 + 6) = & \ 10 + 6 \\ & + 10 + 6 \\ & + 10 + 6 \\ = & \ \underline{30 + 18} \end{aligned}$$

Write this out in the same way for:
(a) $4 \times (10 + 6)$; **(b)** $5 \times (10 + 3)$; **(c)** $4 \times (5 + 2)$;
(d) $5 \times (7 - 2)$; **(e)** $4 \times (a + b)$; **(f)** $4 \times (a + 3b)$;
(g) $3(5a - 2b + 6c)$.

In **(g)** there will be three columns in the stack.

Now we extend this to a ***pair*** of brackets:

We know that:

$$\begin{aligned} 3 \times (10 + 6) = & \ 10 + 6 \\ & + 10 + 6 \\ & + 10 + 6 \\ = & \ \underline{30 + 18} \end{aligned} \quad \text{and} \quad \begin{aligned} 4 \times (10 + 6) = & \ 10 + 6 \\ & + 10 + 6 \\ & + 10 + 6 \\ & + 10 + 6 \\ = & \ \underline{40 + 24} \end{aligned}$$

So, how can we write $3 \times (10 + 6) + 4 \times (10 + 6)$?

There are two ways to calculate it:
1. Since $30 + 40 = 70$ and $18 + 24 = 42$, the answer must be $70 + 42$.
2. But it must ***also*** be $7 \times (10 + 6)$.

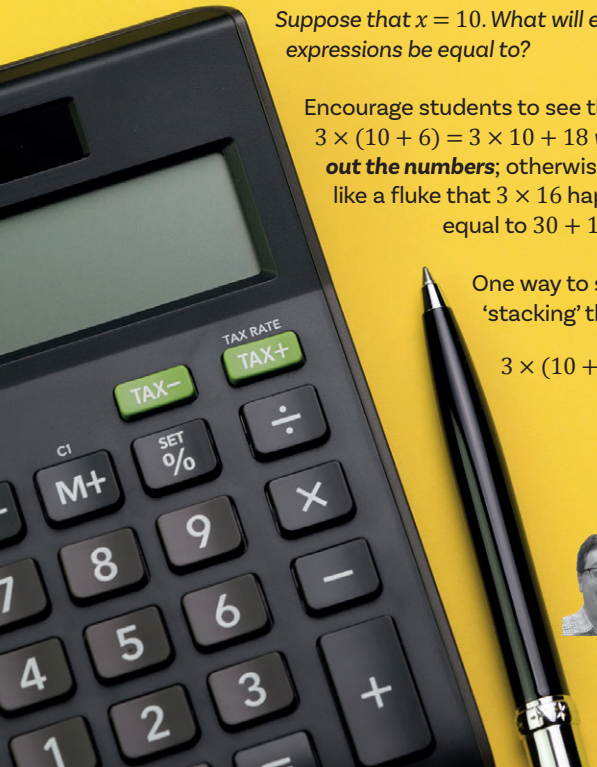
So, we see that $(3 + 4) \times (10 + 6) = 3 \times (10 + 6) + 4 \times (10 + 6)$.
"Three lots of 'ten plus six' plus four lots of 'ten plus six' is equal to 'three plus four' lots of 'ten plus six'."

This is just like $7a = 3a + 4a$.

$$\begin{aligned} \text{So } (3 + 4) \times (10 + 6) &= 3 \times (10 + 6) \quad + 4 \times (10 + 6) \\ &= 3 \times 10 + 3 \times 6 + 4 \times 10 + 4 \times 6 \end{aligned}$$

Write out, in the same way:
(a) $(3 + 5) \times (10 + 6)$; **(b)** $(3 + 4) \times (10 + 3)$;
(c) $(10 + 3) \times (3 + 4)$; **(d)** $(3 + 10) \times (3 + 4)$;
(e) $(5 + 4) \times (5 - 3)$; **(f)** $(5 + 4) \times (5 - 4)$;
(g) $(3 + a)(b + c)$; **(h)** $(a + 2b)(c - d)$.

Checking for understanding
Make up two examples of a pair of brackets expansion; one easy and one hard for each. Include the correct expanded forms.



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