[MATHS PROBLEM]

EXTERIOR ANGLES

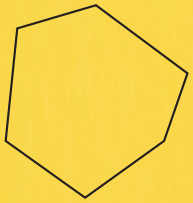
Students often misinterpret the meaning of exterior angles in polygons and how they relate to interior angles

In this lesson, students compare and contrast the correct and incorrect understandings of exterior angles

THE DIFFICULTY

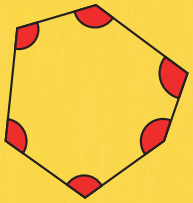
This task is intended to bring to the surface possible confusions over exterior angles:

Look at this polygon. (see bit.ly/112-ext-angles for a printable version.)

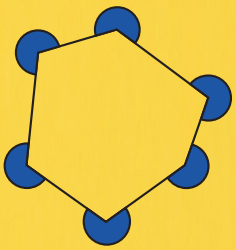


*Clearly mark all the **interior** angles in red.*
*Clearly mark all the **exterior** angles in blue.*

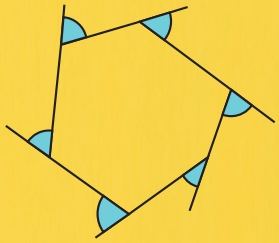
Students will probably correctly mark the interior angles as shown below:



But some students are likely to ***incorrectly*** mark the exterior angles in the following way...



...rather than the correct answer, as shown here:



THE SOLUTION

The light blue exterior angles are sometimes called ***turn angles***, because they are the angles that you would turn through if you were walking around the polygon.

How are the red, dark blue and light blu e angles related to each other?

If students find this difficult, they might benefit from using protractors to check their ideas, or from making drawings of different polygon examples to check that their conjectures work.

Every light blue exterior angle is ***supplementary*** to the red interior angle at the same vertex:

red + light blue = 180° at each vertex

Every dark blue angle is the reflex angle of the red angle at the same vertex:

360° = red + dark blue at each vertex

Finally, every dark blue angle is a ***straight angle more*** than the light blue angle at the same vertex:

light blue + 180° = dark blue at each vertex

This third equation can also be found by adding together the first two equations (the red cancels out).

Checking for understanding

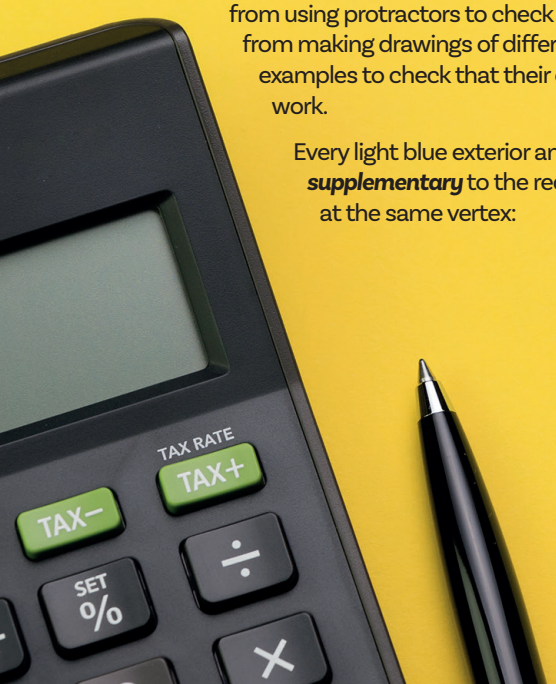
Conclude the lesson by asking students to see if they can write down ***general*** statements about these angles for an n-gon. (Here, with a hexagon, $n = 6$.)

The sum of the red angles = $180(n - 2)°$

The sum of the light blue angles = $360°$

The sum of the dark blue angles = $(360 + 180n)°$

This means that the sum of all of the dark blue angles and all of the red angles must be $(360 + 180n) + 180(n - 2) = 360n°$, which makes sense, because it corresponds to a full turn for each vertex.



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