[MATHS PROBLEM]

FINDING THE *n*th TERM

Students are often confused about how to find the *n*th term of a sequence of numbers, notes Colin Foster

In this lesson, students compare various multiplication tables that are shifted by different amounts

THE DIFFICULTY

Can you find me a linear sequence that satisfies each of these statements?

a. The 5^{th} term is 11
b. The 7^{th} term is 15
c. The 2^{nd} term is 10

*Can you find **more than one** example for each of the statements?*
*Now, can you find me a linear sequence for which **two** of these statements are true? Can you find me a linear sequence for which **all three** of these statements are true?*

By trial and error, students may be able to find examples of some of these, but they will probably not find this very easy!

a and b are true for the sequence 3, 5, 7, 9, ... $(2n + 1)$
b and c are true for the sequence 9, 10, 11, 12, ... $(n + 8)$
a and c (difficult!) are true for the sequence
$9\frac{2}{3}, 10, 10\frac{1}{3}, 10\frac{2}{3}$ ... ($\frac{1}{3}n + 9\frac{1}{3}$ or $\frac{n+28}{3}$)

THE SOLUTION

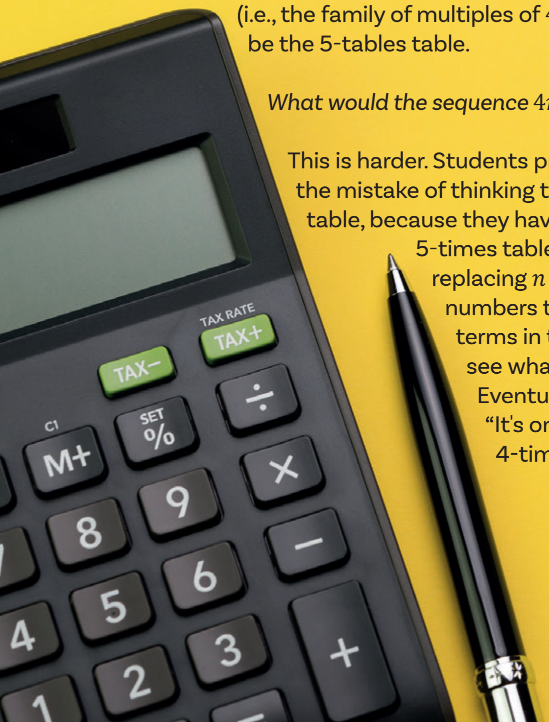
Why do we call this sequence below, $4n$? What would the sequence $5n$ look like?

4, 8, 12, 16, 20, 24, ...

Students should realise that if $4n$ is the 4-times table (i.e., the family of multiples of 4), then $5n$ will be the 5-tables table.

What would the sequence $4n + 1$ look like and why?

This is harder. Students probably won't make the mistake of thinking that it's the 5-times table, because they have just seen that the 5-times tables is $5n$. They could try replacing n by different term numbers to calculate different terms in the $4n + 1$ sequence to see what it looks like. Eventually someone will say, "It's one more than the 4-times table," or "It's the 4-times table, but shifted on by 1". Placing a dot on a number line or graph for each term of the sequence might be helpful.



What would the sequence $4n + 2$ look like?
What would the sequence $4n + 3$ look like?
What would the sequence $4n + 4$ look like?

With these questions, students may say that $4n + 4$ is "The 4-times table again!" – which is correct, except that the first multiple of 4 (i.e., 4 itself) is missing: 8, 12, 16, 20, 24, 28...

Continue asking students:
What would the sequence $4n + 40$ look like?
What would the sequence $5n + 1$ look like?
What would the sequence $5n + 3$ look like?
What would the sequence $5n – 1$ look like?
What would the sequence $5n – 3$ look like?
What would the sequence $–5n$ look like?
What would the sequence $1 – 5n$ look like?
What would the sequence $3 – 5n$ look like?
What would the sequence $\frac{1}{2}n$ look like?
What would the sequence $\frac{1}{2}n + 1$ look like?

We call these sequences 'linear' (or 'arithmetic') because they go up (or down) in a constant amount.

Write a summary of what the family of linear sequences looks like.
What would the sequence $an + b$ look like? Be specific.

We call the expressions that describe sequences "the nth term" because they tell us what the term in the nth position would be equal to.

Checking for understanding
Find the nth term for each of these linear sequences. Start by deciding which times-table (family of multiples) they are related to.

10, 20, 30, 40, 50, 60, ...
13, 23, 33, 43, 53, 63, ...
6, 13, 20, 27, 34, 41, ...
–1, –3, –5, –7, –9, –11, ...
2, 1, 0, –1, –2, –3, ...
$5\frac{1}{4}, 5\frac{1}{2}, 5\frac{3}{4}, 6, 6\frac{1}{4}, 6\frac{1}{2}$, ...

The answers are: $10n$; $10n + 3$; $7n – 1$; $1 – 2n$ (or $-(2n - 1)$); $3 – n$; $\frac{1}{4}n + 5$

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