Lesson plan: Mathematics KS3

FIVE TRIANGLES

Working out which sides and angles can go together enables useful geometrical practice, says Colin Foster

WHY TEACH THIS?

Developing fluency in using trigonometric ratios is important for working with triangles as well as with more complicated shapes

KEY CURRICULUM LINKS

+ Use mathematical language and properties precisely
+ Use trigonometric ratios in similar triangles to solve problems involving right-angled triangles

In this lesson, students are given 15 angles and 15 lengths and have to work out how they can be placed into 5 right-angled triangles. Finding triples of angles that sum to 180° is fairly easy, but sorting out the lengths is harder, and entails using sin, cos and tan as well as some strategic thinking. This lesson provides plenty of opportunities for students to practise using trigonometric ratios to calculate lengths in a context where there is a bit more to think about.

Q How are the angles and side lengths of a right-angled triangle connected?

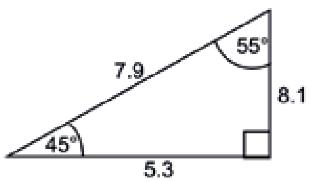
STARTER ACTIVITY

Q Look at this drawing. Can you find some things that are wrong?

There are many things that students could notice:

- the angles do not sum to 180°;
- the side lengths are incorrect, since the hypotenuse should be the longest side, but 8.1 is longer than 7.9;
- the side lengths do not obey Pythagoras' theorem, since $5.3^2 + 8.1^2 = 9.7^2$ (correct to 1 decimal place), not 7.9^2;
- the longer leg (8.1) should be opposite the larger acute angle (55°), not opposite the smaller one (45°).

Encourage students to mention anything else that they notice.



MAIN ACTIVITY

Give students a sheet with five identical, right angled triangles printed on it, along with the instructions shown here (right). A complete task sheet is available at **teachwire.net/fivetriangles**

Five Triangles Task Sheet

The 15 angles and 15 side lengths from 5 right-angled triangles are given below.

Work out which ones go together in the same triangle.

The angles are:
20° 25° 30° 35° 40° 50° 55° 60° 65° 70° 90° 90° 90° 90° 90°

The side lengths (correct to 1 decimal place) are:
0.9 1.8 1.9 2.1 3.0 3.4 3.6 4.7 5.0 5.3 5.4 5.9 6.8 7.7 9.4

Q Can you work out the angles and side lengths of the 5 triangles?

Students may feel overwhelmed by all of the angles and lengths given. If so, they should just try to find some angles that could make up one possible triangle.

It is easiest to try to sort out the angles first. We are told that the 5 triangles are right-angled, but even if we didn't know that we could deduce it. It is not possible for a triangle to have more than one 90° angle, so each of the five 90° angles must go in a different triangle, meaning that all of the triangles will be right-angled. Next, students can put the remaining angles in pairs that sum to 90°: 20°-70°, 25°-65°, 30°-60°, 35°-55° and 40°-50°. So this allocates all of the angles to the 5 triangles.

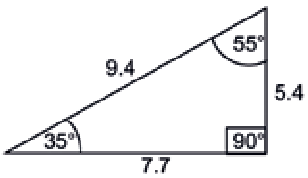
Sorting out the lengths is trickier. Intelligent trial and error is a good approach. One strategy is to begin with the largest length, because we know that the largest length must be the hypotenuse of one of the 5 triangles. The largest length is 9.4, and if we systematically try, say, sin with each angle, starting with the smallest:

9.4 sin 20°

9.4 sin 25°

9.4 sin 30°

we can work out each of these until we obtain a length that appears in the given list (correct to 1 decimal place). In this case, 9.4 sin 35° = 5.4, which is on the list. Then we would try 9.4 cos 35°, which is equal to 7.7, which is also on the list. So it looks as though one triangle could be:



Of course, we could discover later that this triangle cannot be correct, if the other triangles cannot be made from the remaining lengths, but in fact this turns out to be possible. So after crossing out these values, students can continue by taking the largest remaining length as the hypotenuse of another triangle.

However, there are other strategies that will work, so allow students initially to adopt whatever approach they want and see how successful it is. The discussion later on will be richer if a variety of approaches are taken within the class.

Some students may be confused that although the angles have units (degrees) the lengths do not. In pure geometry, lengths are dimensionless numbers, but if students prefer then they could think of them as being measured in centimetres. You may want to emphasise that the 5 triangles printed on the sheet are drawn identically to each other, but these are just sketches, and should not be measured with a ruler or angle measurer.

DISCUSSION

You could conclude the lesson by discussing the students' solutions and how they got them.

Q How did you go about solving this problem? Where did you start? What did you do first? What worked well and what didn't? What was difficult? What was easy?

Students may say that matching up the angles was easy but the lengths were harder. Encourage them to describe the strategies they adopted.

The answers to the 5 triangles (in any order) are:

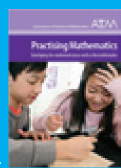
	Smaller acute angle	Larger acute angle	Smaller leg	Larger leg	Hypotenuse
A	20°	70°	1.8	5.0	5.3
B	25°	65°	0.9	1.9	2.1
C	30°	60°	3.4	5.9	6.8
D	35°	55°	5.4	7.7	9.4
E	40°	50°	3.0	3.6	4.7

GOING DEEPER

Confident students could make up a puzzle like this one, by starting with their own set of 5 triangles. Can they make a set where there is *more than one possible solution?*

ADDITIONAL RESOURCES

A nice trigonometry problem is available at **https://nrich.maths.org/2357**



Practising Mathematics is the brilliant new book from the Association of Teachers of Mathematics (ATM). It is full of ideas and activities offering purposeful practice in developing reasoning/ problem solving skills and is receiving rave reviews from top mathematics educators. "A great book" - Colin Foster; and "It's flipping brilliant!" - Craig Barton. Every classroom should have a copy. **www.atm.org.uk/shop/act107pk**

THE AUTHOR

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