

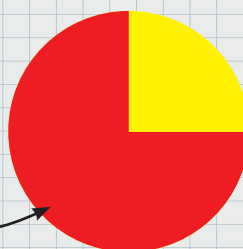
IN A SPIN

When it comes to probability, students may find they come up with the right answer to the wrong question. Activities like this can help them realise their mistakes

Students often get confused about expected values in situations that involve probability and use the idea inappropriately, but there is a way to address this: we can present a task that makes the misunderstanding visible.

THE DIFFICULTY

"Look at this spinner."



"What is the probability of getting red?"

This should be an easy question, and students should answer 'Three quarters'.

"You have to choose red or yellow, and if the spinner comes up with your colour, you win a prize."

"If I spin the spinner 12 times, how many of those times would you choose red?"

Give students time to discuss this in pairs.

The correct answer to the question above is "every time", but some students are very likely to give the wrong answer, by calculating $\frac{3}{4}$ of 12 = 9 and saying "nine times". The expected number of times the spinner would land on red is nine, but this is the wrong concept to use to answer this question.

The expected number is the mean. If you kept spinning the

spinner 12 times, and for each set of 12 spins wrote down how many times it came up red, then, after a lot of 12-spin trials, the mean of those numbers would be very close to nine. But red is more likely on every individual spin, so it always makes sense to go for red. Students often find this counterintuitive.

THE SOLUTION

1. Do it!

A good way to be confronted by this error is to see it happen for real. Get students to make the spinner and try it out, deciding each time which colour they are going to choose. In the long run, students who choose red every time will win more often.

2. Change the fraction

If students say that they would sometimes choose yellow (e.g. three times in every 12), ask them what they would choose for these spinners:



Asking them to think about this may help them to realise their mistake. The third example is likely to generate a lot of discussion.

3. Change the question

The common student response is an example of 'the right answer to the wrong question'. Students need to see that "How many times would you choose red?" is a different question from "How many times would you expect red to come up?". Although we only expect, on average, to get red nine times out of 12, the important thing is that we can't know in advance which nine times this will be. And so, on every spin, it's better to choose red – even if the last nine spins have been red! Changing the question to "How many times would you expect red to come up?" may help students to appreciate the difference.



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