[MATHS PROBLEM]

INEQUALITY SIGNS TURNING ROUND

When solving inequalities, students are often confused when the inequality sign changes direction, says **Colin Foster**

In this lesson, students make sense of when and why the inequality sign turns around

THE DIFFICULTY

Usha is solving the inequality $11 - 3x > 5$.
She writes:

$$\begin{aligned} 11 - 3x &> 5 \\ -3x &> -6 \\ x &> 2 \end{aligned}$$

*How can you tell that her answer is **wrong**?*
***Where** did she go wrong?*

Students may try substituting a value that satisfies $x > 2$ to see whether it also satisfies the starting inequality. In fact, ***every*** value that satisfies $x > 2$ ***fails*** to satisfy $11 - 3x > 5$. For example, if $x = 10$, which is greater than 2, then $11 - 3 \times 10 = -19$, which is **not** greater than 5.

Students may struggle to explain ***where*** Usha has gone wrong, as her steps seem plausible. They might find, by substituting, that her second line is correct, but her third line is incorrect. If they say '*Because the inequality sign switches around when you divide by a negative number*' challenge them to explain ***why***.

THE SOLUTION

What Usha does would be correct for an ***equality***:

$$\begin{aligned} 11 - 3x &= 5 \\ -3x &= -6 \\ x &= 2 \end{aligned}$$

The solution $x = 2$ ***does*** satisfy $11 - 3x = 5$, because $11 - 3 \times 2 = 5$. But it does ***not*** work for an ***inequality***.

*How else could you solve this **equation**?*

An alternative solution method is:

$$\begin{aligned} 11 - 3x &= 5 \\ 11 &= 5 + 3x \\ 6 &= 3x \\ 2 &= x \end{aligned}$$

*Try this method with the original **inequality**.*

This leads to the ***correct*** solution:

$$\begin{aligned} 11 - 3x &> 5 \\ 11 &> 5 + 3x \\ 6 &> 3x \\ 2 &> x \end{aligned}$$

Why does this method work and Usha's method doesn't?

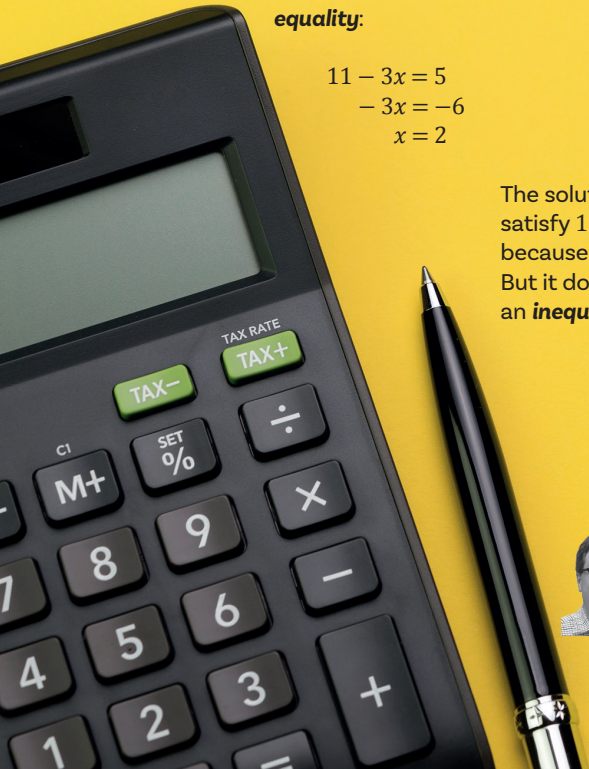
The second method ***never*** multiplies or divides both sides by a ***negative number***. Usha's method does, because she divides by -3. **Dividing or multiplying by negative numbers changes the direction of an inequality**. Students could experiment with a (true) inequality, such as $14 > 10$; dividing both sides by -2 produces $-7 > -5$, which is false.

A further challenge could be to solve this inequality, both correctly and incorrectly, and to explain the difference:

$$\frac{1}{x} > \frac{1}{2}$$

Taking reciprocals of both sides, to obtain $x > 2$, is invalid, since, for example, $\frac{1}{3} > \frac{1}{2}$ is false. Multiplying both sides by $2x$, to obtain $2 > x$ is valid only if $x > 0$. However, we know that if a fraction is greater than $\frac{1}{2}$ then it must be positive, in which case the numerator and the denominator must have the same sign, so $2 > x$ is correct.

Checking for understanding
Make up three inequalities that could catch someone out! Solve each one correctly and incorrectly, explaining why the incorrect solutions are incorrect.



Colin Foster (@colinfoster77) is a Reader in Mathematics Education in the Department of Mathematics Education at Loughborough University. He has written many books and articles for mathematics teachers. foster77.co.uk, blog.foster77.co.uk