Lesson plan: MATHS KS4

KNOWING THE UNKNOWNS

Simultaneous equations that have integer solutions offer a rich context for exploration, says **Colin Foster**

In this lesson, students explore a set of two-variable linear equations, constructed so that when you solve any pair of them simultaneously you get solutions which are all non-negative integers. After students have done this, their task is to construct their own set of 2, 3, 4 or more equations with the same property. This generates lots of opportunity to practise solving simultaneous equations, while at the same time focusing their attention on how the process works and what they need to do to obtain integer solutions.

Q When do you get integer solutions to a pair of simultaneous equations?

WHY TEACH THIS?

Simultaneous equations are a powerful way of modelling situations in which there is more than one variable.

KEY CURRICULUM LINKS

+ solve two simultaneous equations in two variables algebraically

STARTER ACTIVITY

The aim of this starter is to set the scene for the lesson and verify that students can solve pairs of linear simultaneous equations. During the lesson, they will practise this technique and get better at it within an interesting context.

Q Can you solve this pair of simultaneous equations?

$y - x = 2$
$2y + x = 10$

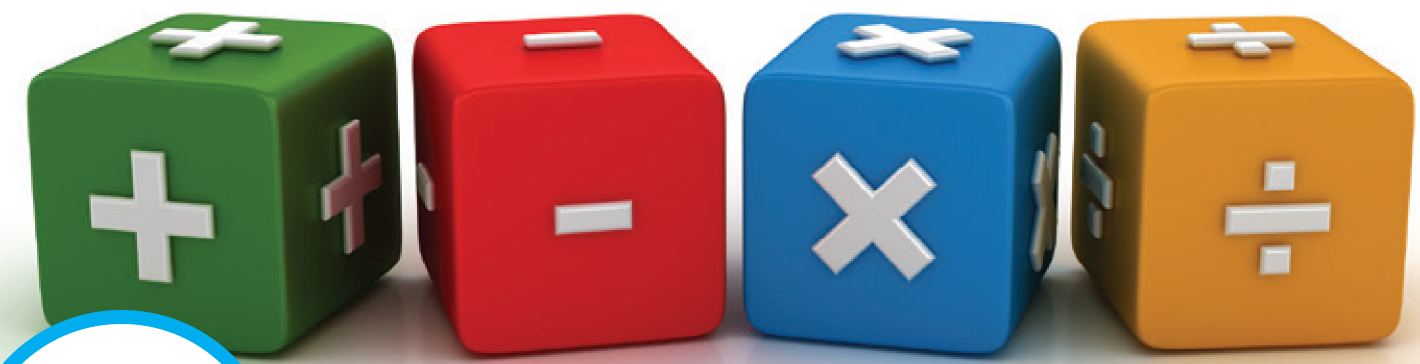
If students can do it, see if they can do it in ***more than one way***. There are at least three possible approaches:

1 add the two equations

2 double the first equation and subtract it from the second

3 rearrange the first equation to make y the subject and then substitute it into the second equation.

Ask students to share their methods and discuss the pros and cons. Make sure that students remember that they need work out ***both*** unknowns (x and y). They should obtain the solution (2, 4).



MAIN ACTIVITY

Q ***Now I'm going to give you two more equations:***

$x + y = 8$
$2y - x = 10$

I want you to solve every equation simultaneously with every other equation.

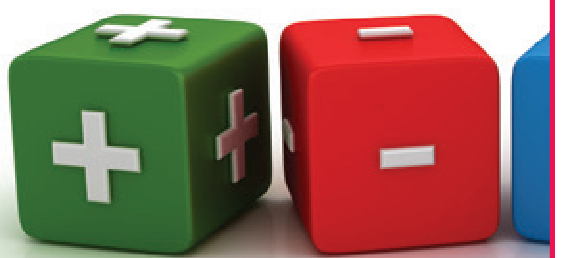
Students will need to think a bit about how many pairs of simultaneous equations they will need to solve. With a total of 4 equations, each can be paired up with 3 others (4 × 3), but this counts each pair ***twice*** (both ways round), so there are actually just $\frac{4 \times 3}{2} = 6$ pairs of simultaneous equations to solve – and one has been solved already, leaving 5.

Students will need to be organised to keep track of which equations they have paired with which equations, as it is very easy to get muddled up. A table might help, or labelling each equation with a letter (A, B, C, D).

All of the solutions are non-negative integers (see the table below), so this is a quick way to check that students are on the right track.

(If you wish to deal with non-integer solutions, a fifth equation, $y + 5x = -4$, could be added to this set, which gives solutions with the others that are ***not*** all integers and ***not*** all positive.)

When students have completed this task, their main task is then to create ***their own*** set of equations (two, to begin with, then three, four, …) so that simultaneous solution of ***any*** pair of them gives integer solutions. They will need to use some careful trial and error to do this. They might find that graphing their equations helps. Students may try beginning with the solutions and constructing equations around them.



DISCUSSION

Discuss how students got on and how they went about it.

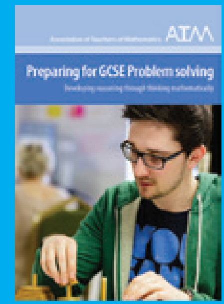
Q *What solutions did you get to the equations I gave you? How did you go about it? For which pairs of equations did you use elimination? Did you use substitution for any of the pairs of equations? Why/why not? Did anyone solve that pair in a different way?*

The solutions are given in the table below:

	$y - x = 2$	$2y + x = 10$	$x + y = 8$	$2y - x = 10$	$y + 5x = -4$
$y - x = 2$		(2,4)	(3,5)	(6,8)	(-1,1)
$2y + x = 10$			(6,2)	(0,5)	(-2,6)
$x + y = 8$				(2,6)	(-3,11)
$2y - x = 10$					$(-1\frac{7}{11}, 4\frac{2}{11})$

Q *What equations did you invent? How did you choose them? Who has a pair of equations that they invented that has integer solutions? Does anyone have a set of three equations that works? Can you convince us that all the solutions are integers?*

See if students can present convincing arguments that their equations are correct. Drawing graphs of the equations and looking for whether they cross at integer lattice points can be helpful.



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ADDITIONAL RESOURCE

A related task is available at tinyurl.com/cattortoise

GOING DEEPER

Confident students could see how many equations they can construct so that solutions to every pair of them are integers. Can they get as many as six equations? If students include equations of the form x = constant or y = constant, that makes it much easier, but if that is not allowed then it is quite tricky!

THE AUTHOR

Colin Foster is a Reader in the Mathematics Education Centre at Loughborough University. He has written many books and articles for mathematics teachers (see **www.foster77.co.uk** and **@colinfoster77** on Twitter).