

[MATHS PROBLEM]

LINEAR SEQUENCES

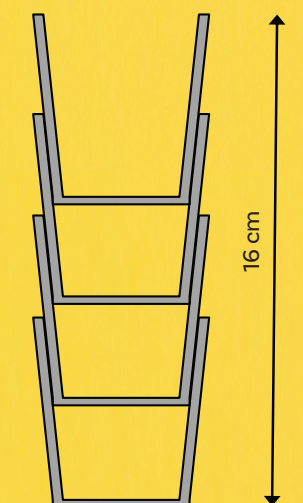
Finding a formula for a linear sequence is often challenging, says **Colin Foster**

In this lesson, students have to find a formula for a sequence based on a physical scenario.

THE DIFFICULTY

Look at this stack of 4 cups. (If possible, have some real plastic cups in the classroom.) It's 16 cm high.

How high would a stack of 8 cups be? Why?

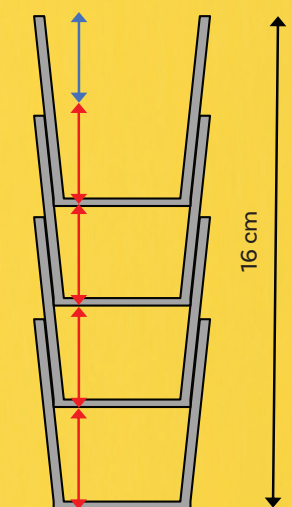


This is a bit of a trick question. Students are likely to double the 16 cm and say 32 cm, but that isn't right. See if students can see why 32 cm is wrong, and whether the correct answer should be more or less than that.

THE SOLUTION

In fact there isn't enough information in this problem to work out exactly how high 8 cups would be. However, we can say that 32 cm must be too large. If we took two stacks of 4 cups and placed them one after the other, that would make 32 cm. But if we slip the second set of 4 **inside** the first set, then that will save some space, so the correct answer must be **less than** 32 cm.

The key thing to notice is that the 16 cm is made up of two different lengths, shown in different colours above. Because of the stacking, there are 4 bottoms (red) and 1 top (blue). (Or you could say 3 bottoms and 1 whole cup. There are many other ways.) What's important is that there aren't 4 equal 'anythings' in the 16 cm – so it therefore makes no sense to just double the height for 8 cups.



How many red and blue lengths would there be for 8 cups?

Following the pattern, there would be 8 reds and 1 blue. When we doubled the 16 cm to get 32 cm, that would be equivalent to 4 reds and 1 blue, **twice**. In other words, 32 cm would be 8 reds and **2** blues. But actually, for 8 cups, we want 8 reds and **just 1** blue. So 32 cm is too long, and the correct answer must be less than this.

To work out the actual answer, we need more information.

Suppose we know that the total height of 10 stacked cups is 34 cm. That's now enough information – see if you can do it!

10 cups will be 10 reds and 1 blue, which is exactly 6 reds more than 4 cups. Since $34 - 16 = 18$, those 6 reds must correspond to 18 cm, meaning that 1 red must be 3 cm. It follows that a blue must be 4 cm.

Now that we know how much a red and a blue are each worth, we can find the correct answer for 8 cups: 8 reds and 1 blue make a total length of $8 \times 3 + 1 \times 4 = 28$ cm. This is shorter than 32 cm, as we deduced.

Checking for understanding

Can you invent a similar problem to this one, that has a nice neat answer, like this one did?

Students will need to think of a related scenario. One possibility could involve shopping trolleys being pushed together into a long line at the supermarket.



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