[MATHS PROBLEM]

MIXED NUMBERS WHEN COMBINING FRACTIONS

When calculating with fractions, students are often confused about how to handle mixed numbers, says **Colin Foster**

In this lesson, students contrast how mixed numbers behave in different situations

THE DIFFICULTY

Enaiya is calculating $4\frac{2}{5}+1\frac{3}{4}$. She adds the $\frac{2}{5}$ and the $\frac{3}{4}$ and then adds on the 4 and the 1. Is this method correct?

*Would it work if the two numbers were **subtracted**, instead of added? Would it work for multiplication? Would it work for division?*

Enaiya's method is correct for addition, and she obtains $5\frac{23}{20}$, which simplifies to $6\frac{3}{20}$.

For subtraction, she tries $\frac{2}{5}-\frac{3}{4}=-\frac{7}{20}$, which might be challenging if students aren't confident with negative numbers – but it does work: $3-\frac{7}{20}=2\frac{3}{20}$.

For multiplication and division, separately multiplying (or dividing) the $\frac{2}{5}$ and the $\frac{3}{4}$, and the 4 and the 1, will **not** give the correct answers.

THE SOLUTION

Mixed numbers are an abbreviated way of writing numbers that contain both integers and fractions.

Just as 42 means $40+2$,

$4\frac{2}{5}$ means $4+\frac{2}{5}$.

The plus sign is ***implied*** in both cases.

Whenever we're unsure, we can write out mixed numbers as sums to help us see how they behave.

$$(4+\tfrac{2}{5})+(1+\tfrac{3}{4})=5+(\tfrac{2}{5}+\tfrac{3}{4})=5\tfrac{23}{20}=6\tfrac{3}{20}$$
$$(4+\tfrac{2}{5})-(1+\tfrac{3}{4})=3+(\tfrac{2}{5}-\tfrac{3}{4})=3-\tfrac{7}{20}=2\tfrac{3}{20}$$

This shows us that Enaiya's method works for both addition and subtraction. But, for multiplication:

$$(4+\tfrac{2}{5})\times(1+\tfrac{3}{4})=(4\times1)+(4\times\tfrac{3}{4})+(\tfrac{2}{5}\times1)+(\tfrac{2}{5}\times\tfrac{3}{4})$$

This is quite complicated, and it is easier instead to write the two mixed numbers as ***improper*** fractions (i.e. as fractions greater than 1):

$$\tfrac{22}{5}\times\tfrac{7}{4}=\tfrac{11}{5}\times\tfrac{7}{2}=\tfrac{77}{10}$$

We could convert this to $7\frac{7}{10}$, if we wanted to get back to mixed numbers.

Improper fractions are also easier when dividing, and the answer comes to $\frac{88}{35}=2\frac{18}{35}$.

Students may conclude that it's ***always*** safer to turn mixed numbers into improper fractions, even when adding and subtracting. This will always give the correct answer, but it won't always be the easiest way. You could ask students to invent cases where it's much easier to use mixed numbers, such as $123\frac{1}{13}+27\frac{3}{13}$.

Note that if students are using calculators to check their answers, they ***must*** use the ■ ▪/▪ button to enter mixed numbers, otherwise the calculator may interpret, for example, $4\frac{2}{5}$ to mean $4\times\frac{2}{5}$, instead of $4\frac{2}{5}$.

Checking for understanding

Make up two mixed numbers, and add, subtract, multiply and divide them correctly. Choose your numbers so that the calculations are about as challenging as with $4\frac{2}{5}$ and $1\frac{3}{4}$

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